Using Student Work to Support Professional Development in Elementary Mathematics

A CTP Working Paper

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Abstract

It is commonly argued that teachers need ongoing engagement with ideas about student reasoning, pedagogy, and subject matter if they are to make sense of the complex demands of current reforms in mathematics education. Drawing on similar arguments about the potential benefits of using student work to organize professional development, this study charts the development of one teacher workgroup over a year. The analysis addresses two questions: (a) How did teachers' talk about student work develop? and (b) What kinds of mathematical and pedagogical issues were raised as a result of their ongoing and changing talk? The study locates teacher learning in their interactions with one another in the workgroup. In monthly cross-grade meetings teachers brought and discussed student work that was generated by a similar mathematical problem posed to students in each of their classrooms. We document the teachers' efforts to detail their students' reasoning and discuss how their engagement with mathematical and pedagogical concerns created opportunities for teacher learning. We discuss the implications of this work for organizing teachers' collective deliberations about student reasoning and pedagogy.

INTRODUCTION

Current discussions about professional development stress the potential advantage of embedding professional education in aspects of teachers' practice (Ball & Cohen, 1999; Cochran-Smith & Lytle, 1999; Lampert & Ball, 1998; Little, 1999). Ball & Cohen (1999) remind us that, "Centering professional education in practice is not a statement about either a physical locale or some stereotypical professional work. Rather it is a statement about a terrain of action and analysis that is defined first by identifying the central activities of teaching practice and, second, by selecting or creating materials that usefully depict that work and could be selected, represented, or otherwise modified to create opportunities for novice and experienced practitioners to learn" (p. 13). Organizing teacher learning around the study of artifacts of practice, such as student work, is one particular way in which professional development can be situated in practice.

Little (1999) contends that "one of the most powerful and least costly occasions of teacher learning is the systematic, sustained study of student work, coupled with individual and collective efforts to figure out how that work results from the practices and choices of teaching" (p. 235). At the same time, Wilson and Berne (1999) argue that we need to better understand how and why such professional development practices might be beneficial to teachers and students. This study examines how one group of teachers, in particular, engaged in studying their own students' work. It addresses two questions: (a) How did teachers' talk about student work develop? How are those changes indicative of a learning trajectory? and (b) What kinds of mathematical and pedagogical issues were raised as a result of their ongoing and changing talk? We discuss the implications of our work for supporting teachers' professional growth.

Background

It is now commonly argued that teachers need ongoing engagement with ideas about student reasoning, pedagogy, and subject matter if they are to make sense of the complex demands of current reform movements in mathematics (e.g., Featherstone et al. 1995, Franke et al., 1998; Hammer & Schifter, 2001; Lord, 1994; Schifter, 1998; Smith, 2001; Wilson & Berne, 1999). A growing body of literature argues that traditional approaches to professional development have disappointing results because they are built on the premise that teachers can be "shown" what it means to center teaching on student thinking. Repeatedly when researchers have studied the effects of such traditional approaches on teachers' practice, they have seen teachers enact curriculum, especially in mathematics and science, in ways that it was not intended, for example, filtering new strategies through their own existing conceptions and knowledge (e.g., Ball, 1990b; Cohen, 1990; Grant, Peterson, & Shojgreen-Downer, 1996; Heaton, 1993). More recent experiments with professional development have focused on "doing" with teachers. In those projects, the rationale has been that teachers need to strengthen their content knowledge and their instructional practices by participating in intellectual communities in which they work on content and pedagogy over a period of time, allowing themselves enough time to develop norms for interaction, navigate tensions and conflict within the group, and build trust to make dilemmas of practice public (e.g., Crespo, 2002; Florio-Ruane & de-Tar, 1995; Grossman, Wineburg, & Woolworth, 2001; Rosebery & Warren, 1998; Stein & Brown, 1997; Stein, Smith, & Silver, 1999).

Research on professional development programs in mathematics such as Cognitively Guided Instruction, SummerMath for Teachers, Developing Mathematical Ideas, and QUASAR have documented the programs' positive impact on teachers' beliefs and knowledge in ways that translated to classroom practice that supported student understanding (Carpenter et al., 1989; Fennema et al., 1996; Schifter, 1998; Simon & Shifter, 1991; Stein, Grover, & Henningsen, 1996; Stein, Silver, & Smith, 1998). These programs provide examples of the need to create intellectual community among teachers in ways that would contribute to the development of a professional knowledge base and ongoing learning. Teachers across the country are also being encouraged to engage in other practice-based professional development programs such as informal study groups, professional networks, and Lesson Study (Crespo, 2002; Crockett, 2002; Featherstone et al., 1995; Fernandez & Chokshi, 2002; Lewis, 2002; Loucks-Horsley et al., 1998; Smith, 2001). Several recent reviews of professional development (e.g., Little, in press; Putnam & Borko, 1997; Wilson & Berne, 1999) as well as large scale studies of teachers' reports on school culture and professional development (e.g., Stupovitz, 2002) point to the glaring need, however, to specify *how* teachers engage in collective analysis. As teachers' involvement in various forms of inquiry groups grows, the need for analyses that show how teachers learn to engage in such collective inquiry increases. In this article, we detail our approach to the use of student work in professional development. We show how teachers' talk changed as they learned to analyze their students' work and what opportunities for investigating mathematics and practice their changing talk created.

Conceptualizing the Use of Student Work

Engaging in the study of student work may be interpreted as one form of what Richardson (1994) calls practical inquiry, "inquiry conducted by teachers to understand their contexts, practices, and their students—to the end of changing practice or increased understanding." As teachers engage in examining their students' work, they consistently investigate what students are doing and thinking and how they have interpreted instructional experiences. To theorize about the power of using student work as a professional development tool, we draw on several discussions of professional education (e.g., Ball & Cohen, 1999; Little, 1999; Wilson & Berne, 1999). Close analysis of student work can provide opportunities for teachers to pause and see ideas in their students' thinking that the day-to-day rush of instruction may not necessarily foster. Discussions of student work allow teachers to raise their own questions about practice and to deliberate about what it is that they want and need students to learn. Such professional inquiry can allow teachers to form generalizations and conclusions from the particular instances of students' reasoning that would guide future interactions in their classrooms. In this way, the study of student work can also stimulate discussions about how and what to teach. By collectively engaging in the study of student work, teachers can make public their own assumptions about teaching and learning and deliberate differences they see in the ways their practices affect students' thinking.

The reasons for advocating teachers' inquiry into student work still seem sufficiently general that it is important to study these ideas in play with teachers (Ball, 1997). Sustained study of student work could be done for an array of goals. Not all teachers who gather to study student work do it or grow in the same ways. It is important to emphasize that our examination of student work reflected particular disciplinary goals. Our work with elementary mathematics teachers aimed to help teachers develop a way of interpreting the various mathematical strategies children used to solve routine problems. We hoped that looking at student work would allow teachers across the grade levels to build a framework for interpreting different mathematical ideas inherent in operating on numbers, particularly ways children come to understand and use their knowledge of the base ten system. Looking at student work, then, would enable us to build ideas about children's thinking in mathematics from the ground up.

We contend that discussions of student work can combine discussions of mathematics, student reasoning, and instruction seamlessly. Launching into analysis of student work provokes teachers to understand the student's particular solution at the same time that it presses them to elaborate the disciplinary knowledge required to make full sense of what the student did. Making sense of children's strategies could be an indirect way for teachers to wrestle with the mathematical issues themselves. Moreover, these discussions can initiate questions about the instructional context in which the work was produced or about pedagogies that can advance the student's thinking. By studying student work in what we call a "teacher workgroup" we expected to encounter enough diversity in teachers' accounts of their practices to surface and deliberate questions about pedagogical goals and methods as well as to deepen teacher's understanding of the mathematics behind students' reasoning.

Specifying Our Model

Our design was influenced by the Cognitively Guided Instruction (CGI) (Carpenter, Fennema, & Franke, 1999 CGI) model in which teachers are presented with research-based frameworks for understanding dimensions of children's reasoning and encouraged to make sense of that information in the context of their classroom. Like another new model called Developing Mathematical Ideas (DMI; Schifter, Bastable, & Russell, 1999), a curriculum consisting of written cases authored by teachers about real events in their classroom, our design asks teachers to make sense of students' mathematical strategies. Our approach differs, however, in important ways from those two programs. First, we engaged in this work during the course of a regular school year, and, second, we provided classroom support in between monthly meetings (neither a necessary component of CGI or of DMI). Second, while the "curriculum" was guided by the CGI frameworks, teachers provided their own "texts" for study. For each monthly meeting, we gave the teachers a mathematical problem to pose to their students, first adapting it to their grade level should they choose. The facilitators played an important role in helping guide and select the materials for the group. There was also flexibility for the work to unfold as we listened to the teachers' conversations and made decisions about which problems teachers should pose to their students for the next meeting.

We deliberately chose to base our work within a school. We met after school for an hour and a half. The school-based approach reflects what many argue is an important way to create an intellectual culture within the workplace. Schools must become places for discussion about practice and principles of learning and productive places for teacher learning if they are to promote student learning (Elmore & McLaughlin, 1988; Fullan, 1991; Hargreaves, 1994; Lieberman, 1988; Sarason, 1996; Tharp & Gallimore, 1988). A model that is built into the regular workday has more promise for being sustained and integrated into teachers' practice than an occasional weekend or summer institute (cf. Lave & Wenger, 1991). The principal supported this integration by giving up one faculty meeting per month for the purpose of our workgroups.

In sum, what sets our approach apart from other attempts to engage teachers in the study students' thinking is the combination of the following set of characteristics:

1. We did not start with workshops or a summer institute that would orient teachers to the ideas we wanted to investigate together.
2. We began with teachers in one school, although voluntarily, who were at different starting points and stages of buy-in about whether this work would be helpful.
3. We held cross-grade meetings.
4. We looked at student work not to score it against a rubric but to explore the mathematical ideas children used in solving problems.
5. Our model, as it unfolded, allowed us to take into account what we knew about the local culture and what would be acceptable to teachers in this school to do together (cf. Goldenberg & Gallimore, 1991)

We list these features not to claim that our approach is superior to others or that it should be the singular approach to professional development. Rather, we believe it is important for teacher educators to carefully detail the characteristics of their work with teachers so that we do not assume that everyone's engagement with student work, written cases, or mentoring, to name a few popular approaches, is the same. Detailing the multiple designs for situating professional development in practice lets us compare our research findings and engage in a productive exchange to understand both the limitations and the promise of varying practice-based approaches.

The workgroups at this school began in 1997 and continue without our facilitation. This paper chronicles the first year of meetings and describes what the practices of the workgroup came to be over the course of that year. It locates teachers' learning in their talk around their students' work, recognizing "face-to-face interaction as a strategic site for the analysis of human action" (Goodwin & Heritage, 1990, p. 283). We follow the work of Rosebery and Puttick (1998) and Grossman, Wineburg, and Woolworth (2001) that claims one way to document what teachers learn collectively is to study how they interact with one another. If teachers are learning new things, we should be able to hear it in their conversations: "knowledge entails skills, ways of talking and interacting, ways of observing and noticing things in the environment and the dispositions toward action and interpretation" (Wilson & Berne, 1999, p.179). Teachers' talk, we claim, should be a key source of evidence about the new ideas they are noticing in their students' work and the questions their discussions with colleagues inspire about teaching and learning mathematics.

Method

The professional development in which teachers participated consisted of two main components: (a) facilitated workgroup meetings centered on students' mathematical work; and (b) observations and informal interactions with teachers in their classrooms. This paper focuses on analyses of the workgroup meetings. In this section, we provide an overview of the participants, our data sources and analyses, and the particular design of the professional development.

Setting and Participants

The study took place at Crestview Elementary School (all names are pseudonyms) in a small urban district. Data were collected during the 1997-1998 school year. It was a distinguished school in the state and was noted in the district for its high math scores—relative to other district schools—on standardized tests. (High scores in this district meant performing at the 30th percentile.) Many teachers viewed the success as a reflection of their focus on developing students' skills and number fact knowledge.

The school's conversion to a year-round calendar in the 1997-98 school year resulted in four K-5 "tracks," each consisting of 13 teachers. At any one time, three tracks attended school while one track was on vacation. The student body, roughly 1,300 students, was primarily Latino (90% Latino, 7% African American, 3% Asian American). The transiency rate was approximately 30%. Over 90% of the student body received free or reduced lunch. Each classroom was bilingual, but students were transitioned to mainly English instruction in the upper grades. All of the teachers in the school were bilingual.

We began the teacher workgroup meetings after two years of building informal relationships with the faculty. Because the year-round calendar necessitates that some teachers are on track, while others are off, teachers met together by track rather than by grade. Four simultaneous cross-grade workgroups were held each month. The work described in this paper involves 11 teachers from a single track in the school. The first author facilitated the monthly meetings among this group of teachers, and the second author attended and participated in the first and last meeting of the year. Each meeting lasted about 90 minutes. The teachers represented a range of grade levels and teaching experience (see Table 1). We met eight times. School administrators and support teachers were also invited to the meetings. The assistant

Table 1. Teachers Who Participated in Workgroup Meetings

Name	Grade	Teaching Experience	New to Crestview
Elena	K	0	Yes
Yolanda	K	0	Yes
Javier	1	14	No
Karen	1	11	Yes
Laurie	1	11	Yes
Sara	1	0	Yes
Juan	1/2	3	Yes
Claudia	2	0	Yes
Beatriz	2	7	No
Alma	3/4	2	Yes
Lupe	4	7	Yes

Note: All names are pseudonyms. First names are used to reflect how we addressed each other.

principal attended three meetings. The principal joined each meeting briefly either at the beginning or the end and spoke regularly with the authors.

Data Sources

Data collection occurred across two settings: the workgroups and classrooms. We documented all of the interactions we had with teachers in the workgroups, in their classrooms and in formal interviews and informal interactions (Franke & Kazemi, 2001; Kazemi, 1999). Over the course of the year, we also held meetings with two colleagues who were working at Crestview to continually plan and reflect on our interactions and experiences with its teachers, children, and administrators. The specific data used in this paper include: (a) workgroup transcripts; (b) teachers' written reflections; and (c) copies of student work shared by the teachers.

Data Analysis

We began data analysis by transcribing the audiotapes from the workgroup and collating the written comments made by teachers at each meeting. We reviewed our reflections throughout the year and, by adding to the beginning of each transcript a summary that captured the flow of conversation, we created a log of what happened during the meeting, noting major themes that emerged from the conversations. For each workgroup, we also listed the contributions each teacher made. The transcribed data were organized into notebooks by workgroup and entered into Atlasti, a qualitative software package, for easy coding and retrieval.

We carefully read through several times all of the workgroup summaries and the original transcripts. During these initial read throughs, we probed the data to see which lent themselves to an understanding of how teachers were talking about student work and what kinds of mathematical and pedagogical issues were raised (Miles & Huberman, 1994). We created two broad categories that reflected issues raised in the workgroup and of interest to this study: (a) understanding student thinking and mathematics, and (b) examining relations between students' mathematical thinking and classroom practice.

We then identified a number of subcategories that consistently emerged and re-emerged across the year creating categories grounded in the conversations that took place (Strauss & Corbin, 1998). These focused codes reflected the topic of conversation.

1) Understanding student thinking and mathematics
   - attention to details of students' strategies
   - discussing students' use of ten as a unit to solve problems
   - discussing aspects of the CGI framework for classifying strategies as they emerged in conversation
   - conversations about the standard algorithm
2) Examining relations between students' mathematical thinking and classroom practices
   - discussing how to elicit student thinking
   - sharing classroom practices
     - Æ ideas about structuring and leading activities

- Æ ideas about helping students solve problems
- extending and building on students' informal strategies
- conversations about standardized testing

We used the focused codes to code all of the workgroup transcripts. The codes were not mutually exclusive. Some segments of conversation had multiple codes. We used the coding process to become more familiar with what transpired within each meeting. We examined the development of talk within each code and relationships between codes. This process allowed us to understand the way teachers' talked about students and whether there were changes or developments in the group's talk over the year. We created analytic commentaries to describe changes in the nature of those discussions. From these commentaries, we created a learning trajectory for the workgroup that captured how issues were raised, deliberated, and related to one another. For example, early in the year "detailing strategies" was linked with questions about how to elicit student thinking. Later in the year, detailing was linked to other mathematical and pedagogical issues. Finally, we selected exchanges that were illustrative of the nature of workgroup discussions and key developments across the year.[1] We used the analytic commentaries to articulate how discussions about mathematics, student thinking, and pedagogy evolved over the course of the year.

Professional Development Approach

Before we present our findings, we describe below in detail the design of our professional development approach, including details of the basic structure of workgroup meetings as well as the frequency and purpose of classroom visits that took place between the workgroup meetings. This description frames our analyses of the actual course of the workgroup meetings.

Workgroups

Our knowledge of CGI frameworks for student thinking guided our interactions with the teachers (see Table 2). We chose this perspective because it makes explicit the principles underlying children's mathematical thinking and is designed to be useful for teachers (Carpenter et al., 1996, 1999; Fuson et al., 1997). Unlike the typical CGI model for professional development, we did not conduct workshops with the teachers in which we explicitly presented the frameworks nor did we design activities using videos or worksheets for them to make sense of how the typologies for problem types and strategies related to one another. Instead, we introduced CGI terminology to the discussion as the teachers noticed different strategies in their students' work. We stressed the influence of different problem types on student strategies as the teachers tried different kinds of problems with their students.

Student work from teachers' classrooms guided the substance and direction of discussions at each workgroup meeting. For each meeting, teachers selected pieces of student work to share with the group based on a similar problem that we had suggested they pose to their class. We encouraged them to adapt the problem by changing the number size and context if they felt the changes would be appropriate for their students. We asked them, however, to keep the structure of the problem the same. The problems were given to teachers individually in between meetings. The group did not work out the problems or anticipate what students might do. At the beginning of each meeting, teachers briefly reflected in writing about the pieces of student work they had selected to share with the group. They also indicated

Table 2. Typology of Word Problems & Strategies, Cognitively Guided Instruction (CGI)

Problem	Direct Modeling	Counting	Derived Facts
Join Change Unknown			
Ashley has 9 stickers. How many more stickers does she need to collect so she will have 17 altogether?	Makes a set of 9 counters. Makes a second set of counters, counting "9, 10, 11, 12, 13, 14, 15, 16, 17," until there is a total of 17 counters. Counts 8 counters in second set.	Counts "9 [pause], 10, 11, 12, 13, 14, 15, 16, 17," extending a finger with each count. Counts the 8 extended fingers. "It's 9."	"9 + 9 is 18 and 1 less is 17. So it's 8."
Separate Result Unknown			
There were 24 children playing soccer. 7 children got tired and went home. How many children were still playing soccer?	Makes a set of 24 counters and removes 7 of them. Then counts the remaining counters.	Counts back "23, 22, 21, 20, 19, 18, 17. It's 17." Uses fingers to keep track of the number of steps in the counting sequence.	"24 take away 4 is 20, and take away 3 more is 17."
Measurement Division			
There are 31 children in a class. If 4 children can sit at a table, how many tables would we need?	Makes a set of 31 counters. Measures out four counters at a time until all the counters have been used. Counts 7 piles of 4 counters and 1 pile of 3 counters. "We need 8 tables."	Skip counts by 4s until 32, "4, 8, 12, 16, 20, 24, 28, 32." Uses fingers to keep track of the groups of four. "We need 8 tables." Skip counts by 4s, 13 times, "4, 8, 12, 16, 20,	"4 times 6 is 24. 7 more to get to 31. So that's 2 more groups of 4. That's 8 tables. But one table only has 3 kids."
Multiplication			
Mrs. North bought 13 pieces of candy. Each piece of candy cost 4 pennies. How many pennies did she spend on candy altogether?	Makes 13 piles of 4 counters. Counts them all up by ones. "It's 52 cents." Makes a row of 17 counters and a row of 8 counters next to it.	24, 28, 32, 36, 40, 44, 48, 52. It's 52 cents."	"10 times 4 is 40. 3 times 4 is 12. 40 + 12 is 52."
Compare			
Rosalba has 17 bugs in her collection. Hector has 8 bugs in his collection. How many more bugs does Rosalba have than Hector?	Counts the 9 counters in the row of 17 that are not matched with the set of 8.	There is no counting analog of the matching strategy.	"8 + 2 is 10 and 7 more is 17. It's 9."

what problem they actually posed to their class and why they made any changes. The reflection focused everyone on their students and served as a basis for discussion.

We then invited teachers to share the variety of strategies that they observed in their classrooms. Teachers could comment on how they adapted the problem, how their students reacted to the problem, and specific ways their students attempted to solve it. As the strategies were described, we recorded them on chart paper so the group could revisit them later in the meeting. We consistently questioned the teachers to engage them in describing the details of the students' strategies. We asked teachers to think of other strategies children could have used that were not yet represented in the conversation, and we introduced other strategies children commonly use if they had not been offered or observed by the teachers.

In order to explore what the strategies revealed about student thinking, we asked the teachers to compare the relative mathematical sophistication that the strategies demonstrated. For example, a strategy that involves counting by ones from 9 to 17 is less sophisticated mathematically than one involving a derived fact, for example 8 + 8 is 16, 8 is one less than 9 so it would be 17. The group organized the strategies into working frameworks that captured the development of student thinking in that particular mathematical domain. The facilitators introduced language from the CGI frameworks to describe the development of strategies (Carpenter et al., 1999). We asked the teachers to articulate the mathematical principles that underlie the various strategies and what the strategies revealed about students' mathematical understandings. We redirected questions about particular strategies and their place within the framework to the group for discussion or encouraged teachers to investigate them when they returned to their classrooms. The working frameworks served as a source for continued deliberation, reflection, and elaboration in subsequent meetings as teachers continued to pose problems to their class and learn about their students' thinking.

After the group categorized the strategies, the facilitator asked teachers to think about what they would do next with particular students to further their mathematical understandings. For example, the group worked on devising questions to ask or problems to pose next. At the end of each workgroup, we invited teachers to try and summarize what we had learned about student thinking during the session.

Classroom Visits

To provide ongoing support to the teachers, build relationships, and collect data, we visited the teachers in their classrooms, typically once and frequently twice, between each workgroup meeting. (We describe this component of the professional development for the reader to understand the entire design, however, this paper does not focus on classroom-level changes in teachers' practice.) The classroom visits helped maintain continuity between workgroup sessions. The visits were not structured formal observations, but rather informal visits. Teachers were able to ask us questions informally. During one of the monthly visits, we gave each teacher the problem to pose for the next meeting.[2] Spending time in the teachers' classrooms provided us the opportunity to learn more about the students' mathematical thinking and the teachers' practices. We did not want teachers to view us as having answers but as having questions and being in a position to continually learn about their students' mathematical thinking. Our informal conversations with teachers consistently focused on their students' thinking and the evolving frameworks that we constructed in the workgroups. We did provide examples, however,

Table 3. Workgroup Problems Given to Teachers

Session	Problem Type	Problem
1	Introduction	Opening discussion about teaching mathematics; no problem posed.
2	Join Change Unknown (JCU)	Ashley has 9 (46)* stickers. How many more stickers does she need to collect so she will have 17 (111) stickers altogether?
3	Separate Result Unknown (SRU)	1. There were 24 children playing soccer. 7 children got tired and went home. How many children were still playing soccer? 2. There were 124 people watching a soccer game. 37 people went home. How many people were still watching soccer?
4	Measurement Division	1. There are 31 children in a class. If 4 children can sit at a table, how many tables would we need? 2. There are 231 children taking a computer class after school. If 20 students can work in each classroom, how many classrooms would we need?
5	Multiplication**	1. Mrs. North bought 13 pieces of candy. Each piece of candy cost 4 pennies. How many pennies did she spend on candy altogether? 2. Mrs. North bought 15 boxes of animal crackers. Each box cost 47 cents. How much money did Mrs. North spend altogeher?
5	Computation**	9 + 7 = _____ 20 + 17 = _____ 28 + 34 = _____ $\begin{array}{r} 29 \\ \underline{+\ 16} \end{array}$
6	Compare	Rosalba has 17 (101) bugs in her collection. Hector has 8 (62) bugs in his collection. How many more bugs does Rosalba have than Hector?
7	Choice of JCU, SRU, or measurement division	1. Yvette collects baseball cards. She has 8 (67) Dodger cards in her collection. How many more Dodger cards does Yvette need to collect so that she will have 15 (105) Dodger baseball cards altogether? 2. Yvette has 34 (274) baseball cards. She wants to put 10 baseball cards in each card envelope (or box). How many envelopes will she need to put away all of her cards? 3. Yvette had 34 (274) baseball cards. She sold 11 (89) of them. How many does she have left?
9	Final Meeting	Overview of all problems and strategies; no new problems posed.

Notes:
* The numbers in parentheses indicate the larger number size we provided.
** Meetings for the multiplication and computation problems were combined into one because the original multiplication meeting was canceled due to a school flood.

through our informal interactions with their students and the questions we asked students during those interactions. On occasion, we suggested additional problems teachers might want to try with their students. We also shared strategies that we observed as we talked with their students.

Rationale for Selection of Workgroup Problems

The workgroup conversations focused around the problems teachers posed to their students. The order in which we posed the problems was not set before we started but unfolded based on our reflections of what was happening in the workgroups. The mathematical domains we chose to focus on during the workgroup reflected those that the teachers were working on in their classrooms such as place value, addition and subtraction, multiplication, and division. Problem types were given in the order shown in Table 3.

We began by asking teachers to pose a join change unknown (JCU) or a missing addend problem. Many adults see the problem as a subtraction problem, but children will often use an adding or joining strategy to find the missing addend. Teachers anticipate that their students who can solve the problem know how to subtract, when actually many students solve the problem correctly without subtracting. We wanted teachers to be surprised by their students' mathematical thinking. Moreover, most teachers across the grade levels were working on addition and subtraction in the first trimester, and we wanted them to consider how the two operations are related since many textbooks separate the study of addition from subtraction. We moved next to multiplication and division contexts. To many teachers, especially in the primary grades, we suspected that the division and multiplication contexts would appear too difficult since their students would not have started instruction in those areas. However, we wanted them to have opportunities to see that all of the children from kindergarten through grade five would be able to solve multiplication and division problems. We would then be able to talk about the power of direct modeling strategies (in which students represent each number and model the action in the problem) and how they cut across mathematical domains.

We returned to the addition and subtraction context but through computation problems as the teachers approached standardized testing time. We wanted to have conversations about how students could continue to use the same strategies that they used in word problem contexts for computational problems. We also wanted teachers to have an opportunity to see how writing problems vertically versus horizontally would affect the kinds of strategies students used. Finally, we hoped teachers would notice that students can use a variety of strategies to solve computational problems. We then moved on to compare problems to help teachers think about how action in the wording of the problem may make a problem more or less difficult for students to solve. In the final two meetings of the year, we revisited earlier problems to have some closure about the principles we had learned throughout the year.

Findings

In what follows, we show how teachers initially talked about their students' work, how the group's normative practices were shaped, and how teachers' classroom practices became an object of discussion. The role of the facilitator in this trajectory is noted throughout the analyses. We do not claim that each teacher experienced the workgroup in the same way. Rather, our analyses follow the discourse of the group as teachers discussed the relationship between their students' mathematical reasoning and their classroom practices.

We have organized our results into six sections that reflect the trajectories of the workgroup.

1. Articulating Initial Ideas about Teaching and Learning Mathematics
2. Forming Norms for Workgroup Practice
3. Surfacing Practices That Inhibit and Promote Student Reasoning
4. Engaging in Mathematics
5. Questioning an Agenda for Teaching Mathematics
6. Temporary Closure

We show how attending to the details in children's thinking was related to teachers' deliberations about classroom practices, their role in the classroom, and an agenda for teaching mathematics. We argue that as the teachers learned to talk about their students' work in particular ways, some teachers experimented in their classrooms with ideas that the workgroup discussions were raising and brought new evidence to the group that provoked further deliberation about mathematics and pedagogy.

Articulating Initial Ideas about Teaching and Learning Mathematics

During our first meeting in September, we did not ask teachers to bring student work. Instead, we asked them to share issues they were struggling with in their classrooms as well as practices that seemed to be working for them. The teachers' articulated ideas that are fairly typical of a group of elementary teachers. Both in the discussion and in teachers' written comments, remarks about teaching and learning mathematics were quite general. A standard textbook series provided the basis for instruction in the overwhelming majority of classrooms. Teachers' statements about what they did in their classrooms suggested some diversity in the group in terms of how they thought about what works in their teaching: the image of a teacher as someone who models what the students are supposed to do (Yolanda and Claudia); a general belief that working together or with manipulatives is motivating (Karen and Alma); a fairly traditional view of engaging students in mathematics by playing addition facts "bingo" or using pre- and post-tests (Javier and Beatriz); and some mention of helping students generate multiple solutions to problems (Juan and Lupe).

The images of their actions in the classroom stand next to the mathematics goals that teachers stated they had for their students. All of the teachers named specific operations, skills, or made global references to problem solving. Only two teachers (Juan and Lupe) mentioned the ability for students to communicate their thinking as a major goal for the year.

During this initial meeting, teachers also shared their frustrations with teaching mathematics. Problem solving was mentioned by all as an area of concern. The teachers nodded with agreement when Alma shared her concerns:

Alma: Right now, I'm concentrating on the basics like 3-digit adding, subtracting with regrouping, multiplication. It's all the basics first but I'm trying to get them to apply what they know about the basics to word problems, have a problem of the week. And so, it usually involves money, and they either have to, you know, figure out what they need to do to figure out to solve it. And they have to include a picture, a paragraph about what they did, and the actual math problem itself. And they're having a really hard time including all three. Or even just deciding what to do to solve it. Like they can tell me what the answer is but they can't explain to me the way they got there. So I don't know if like they've looked at somebody else's paper. So I'm still working on that right now, and that's what I'm really more frustrated with, like taking them through the steps. And even after I've modeled it, and I leave everything right up there. And then I just give them the exact same problem with different numbers. And I still get some who are just—they don't know where to start. So, that's frustrating. (W1: 9/30/97)[3]

Alma's description was similar to what many of the teachers described as happening in their classrooms if they were attempting to incorporate problem solving. How to help students understand what the problem is asking and communicate their thinking would repeatedly become topics for discussion over the next several meetings.

Forming Norms for Workgroup Practice

In this section, we show how the basic norms for detailing student thinking were formed in the workgroup. The facilitator promoted two ways of contributing to the workgroup: (a) teachers should be prepared to share the details of students' thinking and (b) the group's examination of student work should lead to a mathematical focus. What our analyses will show is that the teachers had to learn what it meant to look at student work: they needed to go beyond a judgment of whether the student "got it" or not to an understanding of how the student reasoned about the problem. The facilitator supported this kind of attention by pressing for details and providing suggestions for eliciting student thinking. The group's ability to use the student work as a trace of student reasoning was then related to discussions about students' understanding of number.

"What did the student actually do?"

For the second workgroup meeting, all of the teachers who attended brought student work. The facilitator began the meeting by asking the teachers how their students solved the problem. Most teachers shared general descriptions (e.g., "they used their fingers; they drew pictures") rather than detailed accounts of the strategies students used to solve the problem. They did not offer details that differentiated the strategies unless they were probed, and the probes revealed that, except in one case, the teachers had not observed how the students had solved the problems. Teachers also noted that many of their students were unable to solve the problem.

The two exchanges below illustrate how the teachers inferred what their students must have done without having seen them solve or explain their work. In the first exchange, Claudia and Juan were examining work from Claudia's classroom when they noticed that one of her second graders, Ysenia, had written "9 + box = 17" and then filled in the box. The facilitator asked Claudia to explain how she thought Ysenia had filled in the box.

Claudia: Let's see. [reading Ysenia's explanation in Spanish]. "We need eight more to complete 17."

Facilitator: Do you know how she figured it out?

[There is some discussion about the possible ways Ysenia may have figured it out, e.g., whether she counted up, made an initial set of nine and then counted up; what kind of materials she used, or whether she knew the answer.]

Claudia: Knowing Ysenia, she probably knew it and she didn't have to figure it out. (W2: 11/18/97)

During another exchange, Laurie explained how she had modified the problem to relate to a class unit and lowered the number size because her first graders had not yet worked with numbers above 10. She posed the problem, "Mrs. L has 10 ants. There are 3 grains of food. How many more does she need to feed all her ants?"

Laurie: All my students used pictures. Some, a very small percentage, were able to get it immediately. One kid, put a little grain of food in each mouth, and then counted up from there.

Facilitator: Okay, did he start with the first three or did he count all ten first?

Laurie: I am not sure.

Facilitator: I noticed that he had put numbers. One through ten are labeled on his picture. I wonder when he wrote the numbers.

Laurie: I didn't see him solve the problem, so I am not sure. But knowing him, my guess is that he drew all ten first. (W2: 11/18/97)

These exchanges exemplify the tone of teachers' first attempts to describe their students thinking. The teachers' comments revealed an assumption that the work would speak for itself. The facilitator's role in these exchanges was to probe and wonder about the details, contributing to an expectation that the teachers could have more information about how the students solved the problem.

"How do I understand what my students did?"

We learned, during the course of this second meeting, why teachers had such a difficult time describing their students' reasoning. The teachers relied on what students' wrote and had not engaged their students in conversation about their strategies. Many teachers had posed the problem in ways that inhibited their ability to talk to their students—they gave the problem as a test, sent it home for homework, included it in plans for a substitute, or assigned it as independent work. Despite these choices, teachers still worried aloud that eliciting student thinking was a difficult practice. Not only did they notice many of their students were unable to solve the problems they posed, the teachers felt their students were not good at explaining their thinking clearly.

Noticing teachers' uncertainties, the facilitator broke up the discussion of students' strategies to briefly generate ideas for how to elicit students' reasoning. The facilitator first tried to draw on what teachers were already doing. When she asked for suggestions from the group, only Lupe and Laurie offered ideas.

Lupe: I ask questions like, "Does that make sense?" Because today one of the things they were having a hard time with was that they were coming up with numbers that did not make sense. Like they were saying, if you used the 9 and 17 example. They were adding 9 and 17. I was saying, "How many do you want to get to? You want to get to 17, right. Okay, does your answer make 17? How many do you have now?" And they had a much higher number. So, I ask them a bunch of number sense questions.

Facilitator: Other questions that anyone asks that you find helpful to get at what the kids are doing? Because it's certainly a hard thing—something they need to learn to do, to articulate their strategies.

Laurie: They're not used to really explaining their thought process. I don't think that they really have an understanding that what they are thinking in their head is something that they can verbalize. So I usually say, "What were you thinking of in your head?" You know, "What were the words that were in your head?"

Facilitator: Yeah, 'cuz they'll say that a lot. "I just knew it!" "I did it in my head." Any other questions that you ask? (W2: 11/18/97)

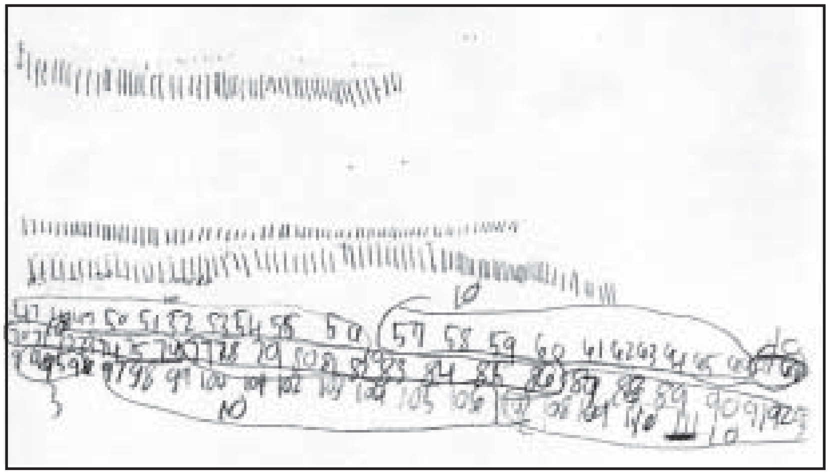
The facilitator's invitation for further suggestions was met with silence. She provided a few additional suggestions—"What numbers were you thinking of? What number did you think of first?"—before the conversation returned to other strategies. She cautioned teachers that questions such as, "Did you count up?" may direct students to adopt a particular strategy that they may not have used. Instead, she recommended that if children say they used sticks or used their fingers, a question like, "Can you show me how you used the sticks or your fingers?" could help elicit the child's strategy more accurately. The facilitator's moves here are deliberate in helping to promote a certain kind of teacher-student interaction in the classroom.

"Look at all those tallies."

Shaping a mathematical agenda during the workgroup meetings was an important goal the facilitator considered from the outset. During the course of the second workgroup meeting, 19 different strategies were shared and listed on chart paper (3 direct modeling, 3 counting strategies, 2 recall, 1 invented algorithm[4], 2 ambiguous, 4 incorrect). Of those, five were offered by the facilitator because the teachers did not volunteer them: (1 direct modeling by tens, 1 counting by ones, 1 counting by tens, 1 derived fact, and 1 invented algorithm). What is important to notice here is that a third of the strategies shared by teachers were either incorrect or ambiguous. And the teachers did not share any strategies that were more sophisticated than counting strategies.

It was during this second meeting that the facilitator initiated what would become a bigger focus for the group, the idea that place value was not just about identifying columns. Rather, it meant being able to understand and operate with "ten as a unit." The facilitator first raised the concept of understanding tens by sharing a strategy she had seen in Alma's third- and fourth-grade classroom. The problem stated, "Ashley has 46 stickers. How many more does she need to collect to have 111 altogether?" Because Alma could not attend the meeting, the facilitator described a strategy she had seen Michael, a third grader, use in her class. Michael had counted up by ones from 46 to 111 (see Figure 1). To keep track of his count, after a few attempts of making tallies, he had written each of the numbers as he counted them: 47, 48, 49, 50, 51, all the way to 111. To get the answer, he counted the tallies or the numbers, but he reached a different answer each time. During class, the facilitator had suggested that Michael try circling each set of ten numbers to see if that would aid his count. This move was shared to provide an example of how teachers could help students build on a strategy they were comfortable with. It is fair to say that the teachers were surprised, perhaps shocked, to see a third-grader struggle so much to find the difference between 46 and 111.

Figure 1. Michael's four attempts to count up from 46 to 111 by using tallies and numbers.



None of the teachers had seen students use a strategy that made use of tens, so the facilitator asked teachers to generate a solution strategy that made use of base ten blocks and to think about how students can advance from counting by ones to counting by tens. In the midst of this discussion, the facilitator offered additional counting-by-tens strategies and one invented algorithm that students could use to solve the problem. These examples highlighted the concept that strategies develop from less to more efficient use of tens. Note too the facilitator's ability to draw on her own observations and interactions with students in teachers' classrooms.

"This is what my student told me."

Teachers came to the third meeting with a much better sense of their students' solutions. The level of detail they provided in this workgroup meeting compared to the last demonstrated more attention to student thinking. For example, Claudia described not only her student thinking but also her interactions with a student who was solving the following problem: "There were 24 children playing soccer. 7 children got tired and went home. How many children were still playing soccer?"

Claudia: I have two students in the same table. One did it exactly like that. I quoted exactly what he said. "I counted back from 24 until I had 7 fingers down. And then I knew I had 17." And then another student that was sitting right next to him did exactly the opposite way. She counted up after 7. And then she said she already knew she had 10. So she said, "Okay, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17." She knew she had 10, and then, um, she had, wait what 7, 24. 8, 9, 10.....17. So 10. So then she knew she had 7.

Facilitator: So then she kept going after that. 18, 19......24. [Alma joins in the count]

Claudia: She knew she had the 10 down and then she saw 7, 17. She has the concept of grouping 10 and not having to see them and just seeing 7, she knew she already had the 10 from before, so 17. So she counted up. (W3: 12/9/97)

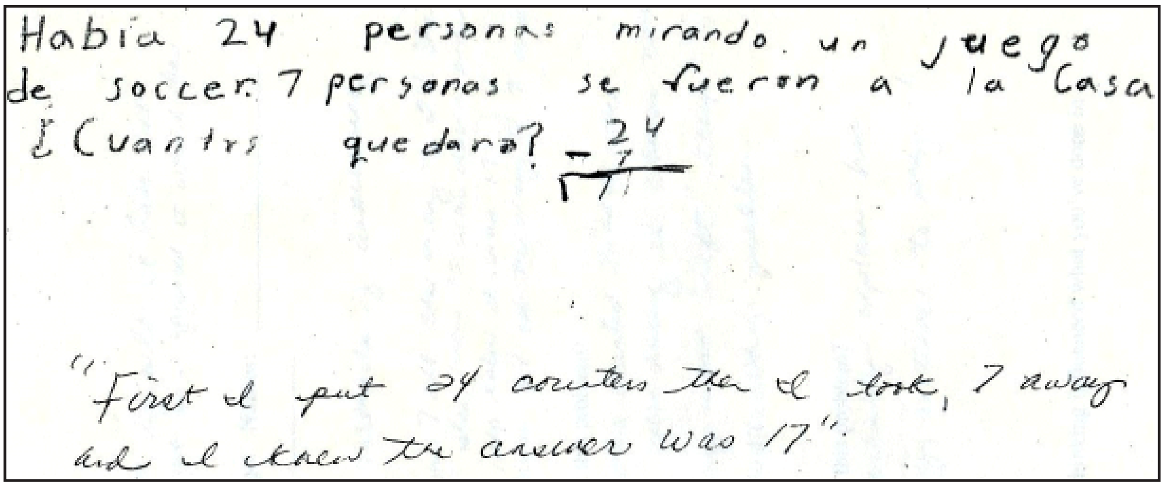
As Claudia described her students' strategies, she read from notes that she had taken in the moment. Two other teachers also came prepared to share their students strategies, using annotations they had made directly on the work itself to aid their memories of what the students had done. This method of keeping track was an idea that other teachers used in subsequent meetings to document their students' verbal explanations or a conversation they had with their students. Claudia, for example, showed the other teachers how she had written her students' verbatim explanations on the back of their papers, something she had not done in the last meeting (see Figure 2).

Ana: I counted up from 7 to 24. I already knew I had ten fingers down so I knew it was 17 when I saw 7 in my hand.

Alex: I counted back from 24 until I had 7 fingers down, then I knew I had 17.

Eric: First I put 24 counters. Then I took 7 away and I knew the answer was 17 (W3: 12/9/97)

Figure 2. An example of Claudia's annotations, capturing how a student explained her thinking. The Spanish reads, "There are 24 people playing soccer. 7 people go home. How many are left?"



Some teachers, when describing a strategy, included a sense of the exchange they had with a student. This is important because more examples became available to the group about how to talk to children about their mathematical thinking. As Alma's explanation demonstrates, teachers observed that a student's written work did not always reflect what the student had done. Picking up on an issue raised in the prior meeting, some teachers felt that their students were not yet comfortable or adept at writing down what they did. In the interaction Alma shares with the group, she comments on the contrast between what she is able to learn from talking to one of her students and what he is able to write down about his strategy.

Alma: I had one boy of mine who usually has a hard time, I mean, he understands math, but it always takes him longer to solve things. And when I came up to him, he told me the answer but he didn't have anything written down. He just had the answer. And I said, "How did you do it?" And he said, "I counted." And I said, "Well, what do you mean you counted?" And so he said, "I put up seven fingers and I counted down 24, 23, 22, 21." And he counted all the way down. He said, "I got my answer." So I said, "Okay, can you write that down in a paragraph for me?" So he tried to, but he didn't explain, I mean he didn't write it the way he explained it to me. (W3: 12/9/97)

Alma's description points to one of the tasks the teachers felt they faced—to help their students translate their verbal descriptions to written ones. What is significant however, in this third workgroup meeting, was that teachers provided more detailed explanations with their initial descriptions, making probes from the facilitator less necessary. Paying attention to the details of students' strategies became a key aspect of sharing strategies in the workgroup context. In later workgroups, if teachers offered descriptions that were incomplete, either the facilitator or other teachers probed for a more detailed explanation.

"I don't even think they realize that they can be grouping them."

During the third workgroup, the teachers mostly reported noticing direct modeling or counting strategies for the separate result unknown problem (SRU; e.g., There were 24 children playing soccer. 7 children got tired and went home. How many children were still playing soccer?). Out of 13 strategies shared, none were invented algorithms, and only one was a derived fact. The upper grade teachers reported that many of their students used the standard algorithm. Some students in Alma's third and fourth grade class continued to use tallies when the standard algorithm proved to be unsuccessful for them. But tallying by ones was still an inefficient and cumbersome strategy. Alma explained that she was trying to help her students see ways of breaking apart the numbers, but the students were not readily applying those skills to subtracting numbers. We discussed ways to help students develop more efficient strategies, much like the discussion we had in the second workgroup around Michael's tallies.

Alma: And this little boy Ricardo…got confused and didn't know what to do. So then he drew 24 sticks, crossed off 7 and then counted. And that's how he did this and then put his answer there [under his notation for the standard algorithm]. Because he was stuck here [working on the standard algorithm] He knew that 4, he told that me 4 is too little. I can't take 7. And I told him, "What do you need to do?" He said, "I'm not sure." He wasn't sure what to do. So I said, "What can you do next then?" So then he just put the 24 sticks, crossed them off and went back and put his answer.

Facilitator: Did you have anyone use the sticks for the bigger one [meaning the problem with the larger numbers, 124–37]?

Alma: I saw that, but most of them were miscounting and they weren't getting the right answer. They would go through and put all 124 sticks and take 37 away. And they would count and wouldn't get the right answer. Because it's too much. They get confused.

Facilitator: So what's going on when they're doing this by ones?

Alma: I don't even think they realize that that they can be grouping them. They're not even realizing that or even just looking at 100, looking at just the 100 and separating the 100 from the 24. They're not even realizing they can do that. And before we went into this, I talked about grouping. We did like grouping tens and looking at a number like that and maybe pulling 100 aside and this left over. We did all of those different things with it, and then I gave them this. But no one did it, no one even tried it that way which I thought was kind of interesting. So, I'm thinking, well, I'll keep working with 10s and 100s now because they really need to understand that to simplify it for themselves.

Facilitator: Maybe we could generate different ways they could group it to solve this problem. What's one way that you could think of? If we wanted somehow to group our tallies or use some other kind of manipulatives?

Alma: They could circle every 10. Like my kids are representing things like using symbols now. Like, they might say, 124 minus 37. They'll take a $100 bill, two tens, and 4 ones. And then they'll try to subtract that way. Or they'll take, the other thing we saw today was they'll take chips or they'll take the little, the unifix cubes and they'll give them an amount [e.g., assign it a value of ten, hundred, etc]. They'll say this is worth this much. And they're doing that.

> Facilitator: Alma, I know from watching your kids. We talked about this last time. One thing that you can do with kids who are making lots and lots of tallies even as the numbers get bigger and bigger, you can ask them, "Can you think of any way of grouping these?" And maybe even suggest, "Can you circle every 10?" After awhile of doing that, you can suggest, "Can you write 10?" And then pretty soon, they'll start by just drawing circles and not have to make the individual tallies and then after that writing 10. So that's one way you can help kids who are stuck. (W3: 12/9/97)

Alma thought that she might have helped her students think of more effective strategies by doing some activities in which they decomposed numbers into their component parts. It is possible that she expected that such "warm up" exercises would seed more efficient strategies. She noticed, however, that students were not using those skills when they were faced with this subtraction problem. The facilitator recognized that students were not likely to directly and immediately use their experiences with Alma's warm-up exercises. In order not to discourage Alma's experimentation but provide her and the group with some possible next steps, the facilitator reiterated the idea that there is a trajectory that students progress through as they learn to use their knowledge of ten to operate on numbers. This facilitative move reflected our intentions to focus the teachers on building on the strategies their students were already using rather than providing them new unrelated ones.

Surfacing Classroom Practices That Inhibit & Promote Student Reasoning

What we have seen so far is the group learning to pay attention to the details of children's thinking and to raise mathematical ideas about the strategies that teachers observed. As the norms for detailing strategies were established and reinforced during the first two meetings, our analyses indicate movement in the trajectory of the workgroup. What we show next is how the process of detailing students' strategies generated evidence that would propel the group to surface classroom practices that both inhibited and promoted student reasoning.

Over the course of the first three workgroup meetings, in the normal course of describing strategies, various teachers in the group shared a range of practices that they regularly used in their classrooms:

- Teaching students the "family of facts"
- Teaching students to put the larger number in their head and count on
- Modeling a specific strategy first so that students would know how to solve the problem
- Teaching key words that signal which operation to use to solve the problem

The teachers shared these practices as a matter of fact, and they did not question them. The facilitator too did not openly challenge these ideas because these practices derive from teachers' convictions and efforts to help their students solve problems correctly. Some of the ideas, such as finding the key words or memorizing a family of facts, were viewed by teachers as "shortcuts" students could take to find the answer. One particular strategy, focusing on key words, emerged as problematic once teachers posed the division and multiplication problems.

To provide some further background about how it became problematic, we need to return briefly to the first two workgroup meetings. In our initial meeting, nearly all of the teachers had reported that they followed what we would characterize as a traditional curriculum sequence, teaching one operation at a time, following the order of addition, subtraction, multiplication, and finally division. During the second workgroup, when teachers had posed a separate result unknown problem to their class, Javier noted that some of his students modeled the problem successfully (12 children playing soccer. 7 got tired and went home. How many were left?) but then wrote "12 + 7." For Karen, that confirmed her practice of teaching addition and subtraction separately.

Karen: This is why we haven't done subtraction yet because…I try to do all the addition before Christmas and then wait until after, just because in the past my students, they got confused. Once you've done addition, I have a little hard time too getting them when you're doing subtraction not to switch back [to addition.] (W2: 11/18/97)

Karen was not alone in focusing on one operation at a time. Only one teacher, Lupe, drew on a variety of operations when posing problems. The practice of focusing on one operation coupled with some teachers' reported practices of focusing on key words caused major problems for students when we asked teachers to try a measurement division problem in the fourth workgroup (e.g., There are 31 children in a class. If 4 children can sit at a table, how many tables would we need?). Although the facilitator did not challenge certain classroom practices directly, she noted them in order to choose subsequent problems that may provoke teachers to question their own practices.

"Do I plus or minus?"

The realization that a focus on key words inhibited their students' thinking came about as teachers noticed that their students were consistently asking them whether to add or subtract without ever thinking through the problem's context. When we moved out of the joining and separating problem types to the division and multiplication problems, some students' blind reliance on key words became starkly apparent. The teachers became frustrated when they found their students used the operation that was consistent with their current focus in class, regardless of what the problem asked. Many students asked their teachers, "Is this addition or subtraction?" "Is this plus or minus?" Teachers reported that some of the upper grade students pulled the numbers out of the problem, tried addition, subtraction, multiplication, and division, and then picked the answer that seemed right. As Sara and Claudia explained their students' attempts at picking the right operation for a problem that involved putting 20 children into table groups of 4, they also alluded to classroom practices that may have contributed to their students' ideas.

Sara: When I explained the problem, they were asking me, "Is this addition or subtraction?" That was it. [*Teachers laugh*]. They had a really hard time. And I kept emphasizing, "There's 20. There's four kids in each table," because I changed the numbers too. And then we used counters. And I separated—here's the 20 kids—there's four kids in each group, asking for them to group them, trying for them to group them. And they still couldn't do that. Until I asked them, "Well, what if you put four kids in one group." And then they starting thinking, "Okay, that's one table." And then they started separating them into each group. But they really had a hard time doing it. But that's what they were asking, "Is this addition or subtraction?"

Claudia: Because that's how they solve all word problems. They look at the key words, "in all," "plus," "how many are left." They're always just keying in on those key words. I have the same problem. I used the original problem, 31 [divided by] 4. But, yeah, they were totally confused at the beginning. And finally I had some that did this, and they represented each person with a little dot. And they circled [every four] and they took the last one. They figured out they had to erase one.

> And then one brilliant student who did four and then she did 4, 8, 12, 16, 20, 24, 28, 31. And then she said, the last table there's only one student. I got rid of one student in the last table. [*Teachers laugh*] But I think that's the problem. I caught myself doing that, always asking them, "Go back and look for the key word. Circle the key word." Then, there's *no* key word in this problem, so that was a problem. (W4: 1/20/98)

In their descriptions of interactions with their students, Sara and Claudia, began to understand the students' confusion and to reflect on how the confusion was potentially linked to their own teaching strategies. Note that Claudia begins her observation by claiming that the use of key words was how "they all solved problems." This way of describing it locates the student as the agent in generating these strategies. As she continues to describe what she sees, her language changes to an assessment that her own agency in students' strategies, "I caught myself doing that, always asking them, 'Go back and look for the key word.'" These observations marked the beginning of the group's attention to this issue.

We returned to the issue of key words in the fifth workgroup when we looked at students strategies for a multiplication problem when teachers were again frustrated that some students added 4 and 13 instead of grouping 4, 13 times. Teachers continued to report that a proportion of their students' first attempts to solve the problem were not meaningful. Students typically pulled the numbers out and added or subtracted to solve the problem. It was important that the teachers noticed the detrimental effect of using a key word strategy in more than one workgroup meeting. The opportunity to address and readdress issues, such as the key word strategy, allowed teachers the opportunity to begin to develop conjectures about the relationship between classroom practices and student thinking. To reiterate a point raised earlier, the teachers in this group had different viewpoints and experiences in their classrooms. Through these conversations, however, practices that seemed to be inhibiting students' reasoning surfaced. Teachers began wondering about the worth of pursuing certain practices in their classrooms.

"They're beginning to get there."

The press to detail and elicit student strategies coupled with ideas about developing students' efficient use of tens to solve problem had encouraged some teachers to begin experimenting in their classroom. In the second meeting, the facilitator had raised the idea that students who understood the problem context could solve the problem by following the action in the problem. After describing Michael's tally strategy (see Figure 1), the facilitator commented on supporting Michael's attempts to use tens as a way of keeping track of his count. By the fourth workgroup meeting, several teachers shared for the first time strategies in which students made use of tens' strategies. The fourth workgroup focused on a division problem for which students figured out how many groups of 20 were in 231. We purposefully suggested a divisor of 20 hoping that a round number would be easier for students to attempt to decompose 231 through more knowledge of place value than simply counting by ones. For the first time, the teachers shared a diverse range of strategies that showed transitions from modeling by ones to modeling and counting by tens. Listed below is the range of student's solutions we recorded:

Marco: tallied by ones in groups of ten to 231. Went back and grouped two tens to make 20 and then counted how many groups of 20 he had. (see Figure 3)

Sylvia: counted by 20s but could not go past 40. When she hit 40, she started counting by ones.

Carlos: drew 231 in hundreds, tens and ones. Then broke down the hundreds into tens. Then circled groups of 20 and counted how many there were.

Others: counted by tens to 231 And then joined two tens to make 20. Counted the number of 20s to get the answer.

Others: counted by 20s to 220, keeping track of the number of 20s used to make 220. Then found the difference between 220 and 231. (see Figure 4) (W4: 1/20/98)

Figure 3. Marco's strategy for solving 231 divided by 20.

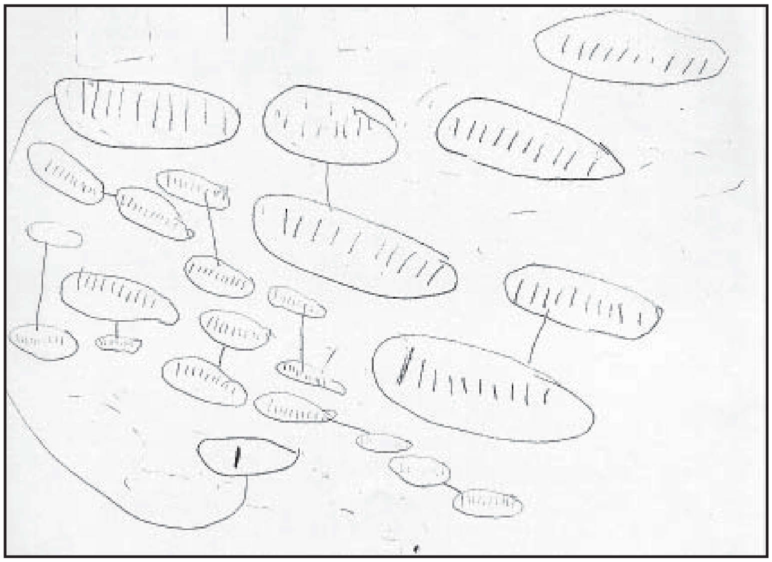
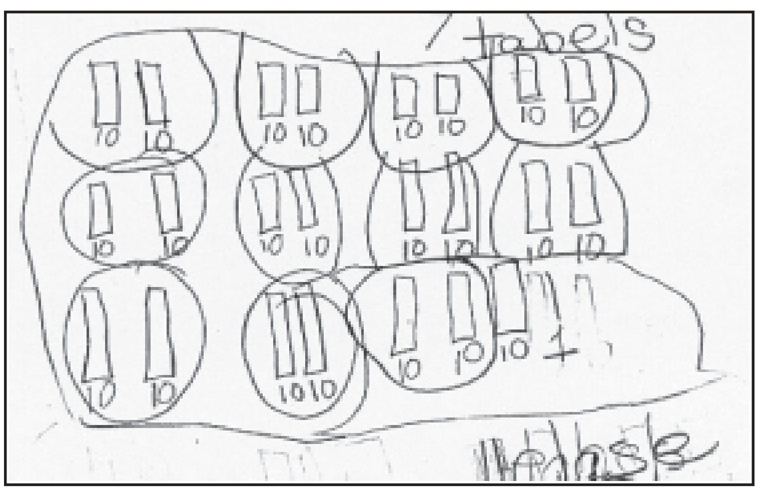


Figure 4. Antonia's use of tens to solve 231 divided by 20.



Teachers discussed how those strategies built on each other. We discussed how students initial efforts might resemble Marcos' strategy of making tallies and then joining groups. We contrasted Marcos' approach with Carlos who built the number from the start by using hundreds, tens, and ones. And finally we talked about strategies like the one in Figure 4 in which the student counted by 20s to begin with, not needing to make all the groups by ones first. As the meeting concluded, Lupe shared Daniel's story, a student in her class who had always struggled with math. He was overjoyed at his tens strategy (the fourth on the above list) and made a series of passionate exclamations when he had solved the problem, "This is the smartest day of my life! I'm finally moving up. I feel famous. I used to be dumb, but now I'm smart. I graduated. I got a handshake." Lupe conveyed the message that students in her class were feeling empowered, and she was seeing them grow mathematically.

Engaging in the Mathematics

Workgroup talk developed further when teachers' experimentation enabled different, more sophisticated traces of student reasoning. In this section, we show how the emergence of sophisticated invented methods from teachers' own classrooms captured the group's interest and in effect engaged teachers more with mathematical ideas.

Until the fifth workgroup, we had not seen any invented algorithms emerge from teachers' own classrooms. Instead, the facilitator had shared one or more possible invented algorithms children could use to solve the workgroup problem. When the facilitator described such strategies, they were just listed on chart paper like other strategies, and teachers did not ask questions about them. Some teachers seemed to recognize their sophistication when ranking them in comparison to other strategies, but they did not intrigue the group. They were not seeing their own students use them. In fact, it was not until the fifth workgroup, as we discussed the division problem, that students made use of tens in a variety of ways, typical precursors of invented algorithms. Those examples came mainly from Lupe and Alma's upper grade classrooms. A few also came from Karen and Laurie's second grade classroom as they reported to the group that they had begun to work more purposefully with tens.

"That is amazing. I'm really impressed."

During the fifth and sixth workgroups teachers reported invented algorithms being generated by their own students for the first time. With this new occurrence, our discussions shifted from detailing the more basic strategies to detailing the strategies that seemed more complicated. Three of the strategies that the third, fourth and fifth grade teachers shared for the multiplication problem (see Table 2) caught the attention of the kindergarten and first grade teachers. Each strategy was explained several times slowly while questions were asked and answered so that everyone could follow the student's thinking. It all started with Karen's interest in Robert's solution, who was a fourth grader in Lupe's classroom. For the problem involving 47 times 15, he multiplied 47 by 5 and then tripled the product.

Facilitator: …So what does this kid understand about this problem?

Lupe: Well, he was able to group it by fives which made it easier for him to figure it out. So…, he's definitely grouping.

Karen: Well, the first thing he did to me, it looks like he took the 15 and broke it into 10 and 5. And then he started with the 47, 5 times. So he knew he could take 15 and make it into you know 47 times 10 plus 5.

Facilitator: And then once he got to here, he's got a partial sum. He's only accounted for 5 47s.

Karen: Actually instead of making that 10 and 5, let's just say he broke it into 5, 5, and 5 is really what he did. I'm just—that is so amazing—I'm really impressed. (W5: 4/28/98)

That day, Lupe shared other invented algorithms her students used. Each made the group pause to figure out how it worked. As we shared students' invented algorithms, teachers shared enthusiastic sighs of "That's amazing!" and "I would never have thought of that." They laughed together as they tried to follow a particular students' thinking. The teachers used their intrigue with these invented algorithms to try out the strategies they saw the students using. For many of the teachers, these strategies and the conversations around them represented a new way to think about doing mathematics. Now, the group was attending to the various flexible ways children used their knowledge of place value to break apart numbers.

The examples from students in their own classrooms enticed teachers to puzzle publicly over the strategies and work to figure out why they worked. For example, in the next to last workgroup, Lupe shared a strategy she had seen Robert, the same student that used the multiplication strategy described above, use in her fourth-grade class. She reported that he often made use of the following invented algorithm. For a problem like 532 minus 199, he would start with the 500 and take away 100 to get 400. Then he would take away 99 to get 301 and finally add back on the 32. We tried Robert's strategy with a different set of numbers, 274 minus 89. Below, we worked together to understand the mathematics underlying it:

Facilitator: If we did it Robert's way, he might do, 200 minus 80.

Lupe: We're getting so stressed out! We can't do this math!

Facilitator: No, you can do it! 200 minus 80. So I've taken away the 80, but I still need to take nine more away, he would say, right? Because he's doing his tens first and then his ones. So I need to take nine more away. That's going to give me 111. The numbers you have to subtract are so easy because you're pulling apart the tens and ones. But I started with 274, not 200, so what do I have to do with the 74?

Laurie: You have to take that away.

Lupe: You have to add it back on.

Facilitator: Ah, ha. Which one?

Lupe: You only used 200 so now you have to use the 74, right?

Javier: You used 80 and you took away the nine also.

Lupe: So you need to use the 74 and the nine – you have to add that back in there?

Javier: Didn't she already use the nine though?

Lupe: [*laughs*]

Facilitator: He's basically changing this to 200, taking away the 80 first and then taking away the nine. I've got that taken care of. But I didn't start with 200, I started with 274. I had more.

Javier/Lupe: So add them on.

Juan: Ohhhhhhh!

Lupe: See aren't you glad you don't teach 5th grade math? [*Teachers laugh*]

Javier: Or you don't have that kid in your class.

Lupe: That's how all my afternoons get!

Laurie: Now you know why I'm not moving up from first [grade]! (W7: 5/19/98)

The teachers' comments indicate that following a student's strategy was not a trivial task for them. In this case, they had to reason through why 74 would be added and not subtracted at the end. Detailing strategies, appreciating the intricacies and possible ways that students could solve problems in invented ways was appropriated by the teachers themselves. Compared to the beginning of the year, participation in the workgroup had shifted towards puzzling over the details of students' invented algorithms and the mathematics they required. Prior to this meeting, the facilitator had offered various invented methods, but the group never focused on them. The teachers appeared more invested in these strategies when they emerged from students in their own school. These strategies pushed their understanding of the mathematics and why knowledge of the ways numbers are composed matter. These exchanges, we propose, helped motivate teachers to think differently about what it meant to understand place value and thus how they could help their students build such flexible mathematical thinking.

Questioning an Agenda for Mathematics

Once teachers began to bring more sophisticated strategies to the workgroup discussions, the workgroup transcripts reveal new discussions about the teacher's role in the classroom. In this section, we show how the teachers' discussion of student work continued to build intellectual community. The group increasingly focused on what teachers could do in the classroom to help students' *build* their mathematical thinking. These conversations, made possible by the group's shared history, contrast with discussions from the beginning of the year when teachers struggled to describe and elicit their students' strategies.

"How do I help the students move?"

The invented strategies came from particular classrooms. From the fifth and sixth workgroup on, Juan, Karen, Alma, and Lupe consistently shared such strategies. Other teachers in the group began to ask them what they were doing in their classrooms that they too could adopt. This is significant in the development of professional dialogue because, as the teachers made practice public, they shared their experimentation with one another. Alma, in the following excerpt, described how she was working with students to develop an understanding of place value, even while she noticed them use the standard algorithm without understanding. Notice that Alma pressed a student to move away from an algorithm that he did not understand and instead use his knowledge of how numbers can be broken up to multiply 13 by four.

> Alma: I asked him about that. I asked him, why did you put the one in that column. He really didn't know, that's just the way you do it, that's the way you do it. So then I said, "Can you think of another way?" We started with the, um, this first one, the 13 times 4 because he did the algorithm right away. And then I asked him, "Okay, could you figure out another way to do it?" And he said, "Okay, I could count by fours, 13 times." So then he put that on here. I said, "Give me another way to do it." And then, he was stuck. So I asked him, well look at the number, at 13. Because we had been going over grouping in 10s and grouping 100s, and just so they could see connections of 10 10s carries over to 100, so they could try to make that connection. So I asked him, "Try to think about what we've been doing, how we can simplify a number. Is there a way to group this number differently?" So then he broke it down into tens and he said, "Well okay if I take 10 and I multiply it times 4 I can get 40." And so he did that, and then I said, "Okay what do you have left?" But I had to walk him through it. I asked him, "Okay, how many ones do you have left? What do you have left?" And he said, "I have 3 ones left." And then he multiplied that by 4 and got 12. So then he added 40 and 12 together and got his answer. (W5: 4/28/98)

Alma, in this example, closely structured her students' effort to solve the problem. She pressed him to use a strategy more efficient than counting four, 13 times, and she supported his initial thoughts by asking questions to guide his thinking. She made public her ways of moving her students away from algorithms they do not understand to ones that they do. The degree of guidance teachers might provide became a question that several teachers raised in the workgroup. Beyond deciding which problems to pose, the number size to use, and anticipating students' strategies, teachers expressed uncertainty about their role in student learning: how much to scaffold, how much to direct students' efforts. The teachers' classroom descriptions, like Alma's, evoked questions about the degree of influence teacher should have in directing the development of students' thinking. The teachers had become more comfortable eliciting their students thinking, but they struggled to think about how to continue the conversation with their students. Alma asked about her role in the classroom during the fifth workgroup meeting. Other teachers agreed with Alma and expressed their concerns about how long it might take for students to develop their strategies and make them more sophisticated without the teachers' explicit intervention.

Alma: ...when we do these things, they're all showing me, everybody is repeatedly showing me the same strategy. So if I want them to move on to something else, is it okay at that point to be able to introduce a new strategy or should I wait until someone, a student, comes up with it on their own. So that they can share it with the class instead?

Facilitator: Well, what do you guys think of that?

[*Teachers all start talking at once.*]

Elena: I'm afraid it's going to take too long.

Karen: You only have, I mean, you've got to move on at some point

Lupe: One of the things that has been working is to ask, what's another way to solve it or could you group this in a different way. You know it's so tempting to say, you know, there's this trick you could do, watch this! [*laughs*]

Alma: I don't know if I should wait that long until somebody finally comes up with their own or if I should like, here's a new one, let's try it! [*laughs*] I mean, I don't know if it's better just to wait to see if they get there on their own. (W5:4/28/98)

Karen and Lupe echoed Alma's concerns later during the workgroup, "I know where I want them to go, and it's how much influence should I have getting them there." "How much should I jump in there? How much should I have them do it?" It is in this way that the teachers' collective inquiry led them to important pedagogical questions about advancing their students' understanding.

"That's not happening in my classroom."

We use a long excerpt from the conversation in the sixth workgroup to illustrate where the teachers' descriptions of their classroom experimentations led the group. In the excerpt, Alma described her students' response to a compare problem, where they have to figure out how many more bugs Rosalba has in her bug collection than Hector. Alma said she was frustrated because some of her students pulled the numbers out of the problem and just multiplied them and were unable to give her a reason why. When she encouraged them to move away from the need to select an operation and pressed them instead to think of ways to follow what was happening in the problem, the students used what she saw as very basic strategies, using "sticks" or base ten blocks to model the relationships in the problem.

Alma: A lot of them were doing that [multiplying] but then they can tell you what they are looking for which really confused me. And then we even did try, don't use any—try not—you don't have to use a symbol, addition, subtraction, multiplication—don't use any of that. We tried that and then I got mostly like sticks, lots of sticks, and then some people were grouping by 5s. I had one little girl who put them in tens. And then I asked her, can you represent this in another way? Instead of using sticks, now what's another way you can represent the same groups of 10. But she never got to; she didn't think of just writing the number 10. She never got to it. So I feel like, do I tell her now? Or do I just let her discover it? And we did do, where someone came up, I think it was Antonia who came up and did her groups of 5. And then, that's the class, is there any other way we could write this? We had, could you do this, and we put a 5 above it, and they said, oh, yeah, you could do that. But no one came up with it. I was amazed. I mean, some, like Mynor is really—algorithms he has down. And he can usually figure out what he needs to do given a word problem, but not even he came up with a way to represent the tally marks in another way.

Facilitator: So what do you guys think of that?

Javier: Well, just like we see that a small number of the students are able to go from the basic to a little bit higher level concept. What I just do, like you saw, you know, I guide them through the questioning process and then see where they are and then not feel guilty about saying, okay then you know, how can we find the answer in a different way. So, I guess that's what I would do and then like you said, how about going back the next day, and sharing the different ways that we got to an answer, and maybe they could learn from that. Or at least we expose them to different solutions. (W6: 5/5/98)

Alma sensed that she was making slow progress toward her new goal of helping her students develop flexible ways of pulling apart and putting together numbers. She looked to the group for guidance about how to help her students generate more mathematically efficient strategies. When the facilitator invited others to comment, Javier expressed his own view that all students may not be able to develop more efficient strategies. Javier told Alma to provide more guidance and not feel guilty about it. As this discussion continued, however, Javier was challenged by Juan who found that modeling solutions for students was not necessarily the best response. In his class, Juan observed that modeling strategies for students actually reduced the diversity of solutions. He explained:

Juan: That's my concern that showing them a certain strategy—because every time they do the problem, they always use the same strategy that I've used [*some teachers around the table nod and say "yes" to show agreement*] and so you know I'll see the same thing in every journal. And that's the hard thing, try to, you know give them different strategies to use and having a variety of strategies to use in their journals. Because I always see the same strategy over and over again.

Facilitator: When you model the strategy for them?

Juan: yeah, yeah (W6: 5/5/98)

Claudia, who had also been trying to engage her students in more open-ended problem solving became interested in what Juan had observed. Below, she asked Juan when he models the solution for students. She had experienced her own frustrations with eliciting students' strategies:

Claudia: So you always model them before you let them do it?

Juan: No, I always model it afterwards.

Claudia: And they remember it for the next day?

Juan: Hmm, hmmm

Facilitator: Is that happening in your class? [*to Claudia*]

Claudia: Well...not really [*quietly*]

Juan: Because we never talk about it before. We, we, I introduce the problem, but I let them go on their own, and then at the end, I usually talk about it.

Claudia: What we do with the word problems is, I put up a word problem, and right after they finish their math pages, it's sort of related or like this stuff [*meaning workgroup problems*]. So, what I notice, and I'm talking about the last problem, the one Mrs. North needs, she's going to buy candy. So, I just put up the problem, and we just read it out loud. And then somebody starts to figure out is it plus, it is take away? And then they're all yelling it out and they're all looking at me to see if I'm going to say yes, or no. And then I say, you know you just need to figure it out. You can work with partners. So they go and they work. And then they all come to the rug, and then I choose 3 people. And I just make little boxes on my paper - on the same paper where I wrote the problem. And I have them come up, and they do exactly what they did on the paper and then they explain it. And they like, some of them write down words. So they're starting to get into that. But I have like different strategies, because especially I don't say anything. Some of them are just trying to do the same thing that is in the math book, you know whatever they did that day in their math book. So like some of them actually tried doing subtraction with this regrouping problem and like double digit subtraction and like regrouping that way but it was wrong. It was actually multiplication. I had three of them, or I had more than three get it right. But I had three very unique ways, and I said nothing to them about how to do it, and they got it right.

Juan: I get that

Claudia: I do agree that sometimes they just try to apply. They don't even want to read into it. They just want to look at the numbers and they just want to go and do it really quick and finish it. So they just use that one symbol we're using for that day or for that week or for that chapter. So, yeah, I do see that, but I give them no guidance for the word problems, so they're on their own. And they know not to come ask me until we're on the rug. Most of them aren't smart enough to know what they did the day before, you know what I mean? They don't remember so... (W6: 5/5/98)

Claudia, who had continued to follow the textbook as the principle guide for her curriculum, inserted what she described as problem solving time during each lesson. After she posed a problem, the students' first reaction was to try and guess the operation, an idea that had apparently remained alive in her classroom. Claudia described her reaction next. She felt that it was important during the problem solving exercises not to give any guidance to her students. And although she noted that some of her students still have trouble with the problem because they try to use the operation they were working on in class, she found that she was still able to elicit at least three viable solutions from the entire class. She offered this as evidence to back up Juan's feeling that modeling solutions first sends the message to everyone to use the strategy modeled by the teacher. Claudia, while still having difficulty engaging all of her students in reasoning through the problem, found that not modeling a solution first for the students produced a diverse set of strategies. Claudia and Juan's observations, in subtle ways, counteract Javier's earlier statement that only *some* students will understand and therefore guiding them through a problem is an appropriate way to help.

Karen, who had been listening to this entire exchange, returned to Alma's original questions and offers her a suggestion. She tells Alma to reduce the number size in the problem and first help the students articulate how they solved the problem, and then increase the difficulty of the problem by changing the number size.

Karen: I've been listening to Alma's question while I've been listening so I'm like okay, what—and I don't know how much time you have, cause this would be a little time consuming, you know, that's the only negative. But what if you gave them a comparing problem like the one you did, the 5 and the 1 because some students would be able to do that. And let them verbalize how they did it. And then gradually make the problem more and more difficult until it's at this caliber.

[*There is more conversation among the teachers.*]

Alma: And just have them verbalize it instead of like even no notating—just talk about it.

Karen: Oh, that's a good question. [*teachers agree*]

Alma: Like just do that instead.

Karen: I was thinking both even though I said verbalizing but maybe, yeah, maybe just let them talk about it. (W6: 5/5/98)

Karen's idea reflected her conjecture that students in Alma's class were getting caught up in the mathematical notation. If Alma were to release the students of the need to decide on an equation and instead concentrate on verbally describing the relationships in the problem, Karen thought she might have a better entry point to help her students. This extended exchange illustrates an important shift in the teacher's interactions with one another. Their comments still reflect various points of view about children's abilities and mathematics. What makes the tenor of this conversation so new is that they are beginning to compare how a particular practice, in this case modeling strategies, is influencing students' reasoning. This marks a shift toward public engagement with one another about their teaching. What they notice in their classrooms is both promising and bothersome to them—their knowledge is incomplete and tentative. Yet, their discussions are creating a press to return to their classrooms and continue to experiment and then come together again to further deliberate about their observations and pedagogical decisions.

Temporary Closure

The workgroup discussions had raised new questions for the teachers. To end the year, the facilitators wanted to have a way to review what the group had learned and accomplished. Thus, for the final workgroup of the year, the facilitators created a handout showing all the problems we posed to the students throughout the year and the range of strategies we documented. For each problem, the facilitators ordered the strategies in the level of sophistication the group had discussed all year long, but labels such as direct modeling, counting, or invented algorithms were not used. During the workgroup discussion, however, the teachers noted that the strategies progressed in sophistication in a similar way regardless of the type of problem. Alma, who had talked a lot about her efforts across the year to help her students understand what the problems were asking for summarized what she saw for the group:

Alma: It seems as if they progress like from simply like directly modeling a problem using tally marks or circles or whatever they choose to use and eventually being able to replace those with symbols whether it's a number or whether it's the base 10 blocks, whatever it is. And then they do that. First I saw, like I saw in my class, first the tally marks, like for everything they did. Then they finally went to maybe then grouping the tally marks into groups of 5. And then they started doing it in 10s and then finally started using the numbers to replace the tally marks. And then some of them have now started just looking at the numbers and saying, okay, that's a ten and that's a ten, let's put those together and we'll put the ones together. And so it seems like to progress to progress from the more basic to where they're actually becoming more sophisticated and understanding the numbers themselves and how they relate. And then I also notice that the old way of thinking math for me because it's the way I learned, it's like okay this certain word or this certain problem just tells me I need to subtract. But I'm noticing now that they're able to solve something by either using, like using addition to figure it out or subtraction or if they understand how to do it, it doesn't matter what operation they use. They're just able to figure it out because they have more of an understanding of how they numbers relate to each other.

Facilitator B:[5] Is that what other people have seen too in terms of sequence? Are there other things that you've seen? That's a pretty nice description of sequence.

Laurie: I'm not seeing quite that level obviously [*teachers laugh*]. But I do see a sequence except that I don't know how to move them to get to the next stage. You know there are two or three in my class that can do it and some of them that have the knowledge of it, you know, can know the number fact and don't need to do it. You know, getting those middle kids to move up there. So I would love some ideas on how to do that. (W8: 6/9/98)

Given the conversations that teachers had in the workgroup, Laurie was expressing what many teachers felt. They had developed the ability to better attend to their children's reasoning but questions remained about how to respond to the actual day to day reasoning that they witnessed, a challenge that researchers have argued teachers must learn to do (e.g., Ball & Bass, 2000). In effect, our study of student work raised many more nuanced questions for teachers to consider together.

Both authors were present for this last meeting, and when the second author asked the group what they thought helped children move along in their strategies, Juan made the following observation.

Juan: I think they have an understanding of what 10 represents and that's what it comes down to in my class that the students that understand 10s are able to solve these problems in an easier fashion than the other students who struggle through these, some of the problems. So it's in my class, it's an understanding of 10s and 1s and 100s. (W8: 6/9/98)

After Juan's comments, the group spent time discussing how teachers were trying to build students knowledge of tens in their classrooms. Because this workgroup was the last one for the year, our conversation reviewed many of the issues that had arisen for teachers across the year: the role of the standard algorithm, using the textbook in alternative ways to push students to think about relationships among numbers, ideas for places to start next year. The teachers voiced their intent to begin the following year focusing on aspects of number that differed significantly from the way they characterized their mathematics program at the beginning of the year. Instead of naming operations, the talk during this final meeting related to the ideas they had explored across the year about place value.

Discussion

We reflect upon the trajectory of teachers' talk in the first year to make several conjectures about the use of student work and the potential opportunities for learning that examining student work opened up for teachers. We also discuss how our analysis speaks to two enduring and stubborn challenges in professional education: overcoming norms of privacy (Lord, 1994) and building a professional knowledge base (Hiebert, Gallimore, & Stigler, 2002).

Teachers Must Learn How to Examine Student Work

Student work, used in the way described in this paper, was a key artifact that provided a way to negotiate meaning among participants in the workgroup (cf. Wenger, 1988 and Wertsch, 1999). We agree with Ball and Cohen (1999) that "simply looking at students' work would not ensure that improved ways of looking at and interpreting such work will ensue," (p. 16). The trajectory of one year's work reveals that the teachers with whom we worked first had to learn to attend to the details of their students' thinking. Even though our meetings were structured from the very beginning to detail students' strategies, teachers did not come to the first meeting prepared to do so. Instead, many teachers assumed that the pieces of paper themselves would tell the story. It was further evident in the way they assigned the problem that many assumed that conversations with students about their solutions were not necessary.

Maintaining the structure of the workgroup allowed us to sustain and promote an emphasis on documenting the details of student thinking. The facilitator played a key role in pressing for these details. To understand what children know about number and operations, teachers must notice how they count, what numbers they start with, how they break apart the numbers and put them back together. A student's answer alone to a problem or a general comment such as, "I put the numbers together," is not sufficient. A press for details allows for a closer look at the mathematical ideas the student is using. We intentionally facilitated the discussions so that teachers would return to the idea of noticing how students were learning to break apart and put together numbers using their knowledge of the base ten structure of numbers. We imagine that without such a focus, meetings might have been interesting but would not have encouraged teachers to follow a particular course of experimentation in their classrooms. Other reports of engaging teachers' in collective inquiry have also emphasized specific content focus as an important contributor to what the group is able to accomplish and to gains in student achievement. (Cohen & Hill, 1998; Garet et al., 2001; Goldenberg & Sullivan, 1994; Kennedy, 1998; Rosebery & Puttick, 1998).

A key outcome of our analysis is that we observed the teachers use of student work shifting early in the practice of the workgroup. In order for teachers to interpret their students' reasoning, they began to use the student work as a trace rather than a complete record of their students' reasoning. Several teachers came to workgroup meetings with their own annotations or they began to recount their interactions with the students, thus explaining how the work was produced and how to interpret it. The act of detailing students' reasoning opened up discussions about practice because teachers began to make public what they were noticing among their own students.

Opening up Opportunities

Teachers' close consideration of student reasoning opens up opportunities to deliberate mathematical and pedagogical questions. Examining student work, as we structured it, allowed teachers to surface their confusions and uncertainties, not just about student reasoning but also about mathematics and classroom practice. The discussions created opportunities for teachers to notice the mathematical ideas students were using. As we noted, this led to the group's engagement with sophisticated computational strategies they were noticing in their classrooms. Our use of student work provided an entry point for teachers to explore mathematical ideas and gave them opportunities to make sense of efficient student-generated algorithms. The need to deepen teachers' content knowledge in mathematics has been aptly demonstrated (e.g., Ball, 1990a; Fennema & Franke, 1992; Ma, 1999). And we observed that making sense of student work provided an indirect way to consider important mathematical issues. We observed, however, that the group did not delve into mathematical issues immediately—it happened only when they noticed strategies in their students that intrigued and puzzled them.

Pedagogical issues related to helping children develop more sophisticated strategies also surfaced once the group saw students in teachers' own classrooms using such strategies. This development was important, we argue, because the belief that invented strategies were anomalies—a belief held by some members of the group—began to be questioned. Getting into the details of student thinking raised important questions for the teachers about their role in helping children generate more efficient strategies. We contend that when teachers engage in ongoing study of student work, it can create a cycle of experimentation and reflection that can contribute to generative learning (Franke et al., 1998). The student work can raise questions teachers can investigate by returning to their classroom. The classroom then produces new artifacts that must be examined and understood, which then leads to further experimentation. Such professional inquiry can allow teachers to form generalizations and conclusions from the particular instances of students' reasoning that would help them plan for and articulate goals for future instruction. We observed teachers make conjectures at the end of the year about the kinds of experiences they would create for their students at the beginning of the following year to continue to build more robust knowledge of number and operations.

Diversity in Teachers' Thinking and Experimentation as a Resource for Learning

Our experience with this group of teachers lends credence to the idea that examining student work is a promising way of beginning to work with a diverse group of teachers. The group was diverse both in terms of grade level taught and of perceptions about teaching and learning. In end-of-the-year interviews, each teacher commented on how illuminating it was for them to meet in groups with teachers from each grade level. Rather than inhibit conversations, the cross-grade aspect of the group composition allowed teachers to see how children's understanding of place value, the particular focus of our work, developed across the elementary years.

Our group was also diverse in the ways teachers reported on their engagement with students in this classroom. Not all the teachers experimented with these ideas in the same way. Through the work they brought to the group, teachers like Lupe, Alma, Juan, Laurie, and Claudia suggested implicitly and, at other times, explicitly the ways they were experimenting in their classrooms. Because the group had multiple ways of relating the workgroup discussions to their classroom practices, the experiences of this subset generated new ideas for the group to consider and from which to make new conjectures for classroom practice. The frustrations they shared in the group also underscored the challenges they faced in helping children articulate and build their ideas. This diversity also became a resource for teachers to compare and question each other's practices. When students performed differently in different classrooms, workgroup conversations opened up for teachers to make their practices public. Our work suggests that teachers do not necessarily need upfront workshop experiences aimed to "put everyone on the same page" before they can begin to study student work. They can learn about student reasoning, explore their own mathematical knowledge, and raise instructional issues in the process of learning how to examine their own students' reasoning. This kind of engagement can create possibilities for shared perspectives.

Future Directions

We compare our conjectures to an emerging set of findings about the use of student work with teachers of mathematics. In two studies of teachers' talk about student work (Crespo, 2002; Crockett, 2002), researchers reached different conclusions about the degree of productive conversations student work evoked. Crespo found that teachers' discussions of their own mathematical problem solving was more critical than discussions of student work. Specifically, when teachers engaged in mathematics together, she found the discourse to be *exploratory*—that is "talk that is characterized by speakers seeking and showing intellectual involvement; explicit disagreements and public disclosure of uncertainties and confusion," (p. 13). In contrast, Crespo found that teachers engaged in *expository* talk when they discussed their own students' mathematical work, "which can be characterized by the use of monologues; speaking seeking and giving approval; and non-analytical or unproblematic narration of events" (p. 13). The opposite was true in the teacher inquiry group that Crockett studied. When teachers engaged in mathematics together, they were more concerned with getting to the right answer, yet when they examined student work (though not their own) in order to score it against a rubric, "the discrepancies in their scores generated conflict that gave rise to arguments about what counts as mathematical understanding" (p. 622). With such a limited body of literature, it is too early to make definitive statements about how and when student work supports critical deliberation among teachers. We did find, in our work, that teachers had to learn to attend to particular features of their student reasoning. Studies that compare different ways to structure, facilitate, and focus conversations around student work are clearly needed.

Endnotes

1 There were four simultaneous workgroups run by project staff at this school. Two of the workgroups were centered on discussions of student work, each facilitated by one of the authors of this paper. The other two workgroups were focused around discussions of written cases highlighting student reasoning, each facilitated by other members of the research team. The analysis described in this paper was applied to both workgroups using student work. The analyses revealed a strikingly similar pattern of change in both workgroups across the year in teachers' discourse. This is not surprising given that the facilitators worked closely together and planned the agenda of the meeting. While further study is need, it suggests that at least at the initial phases of beginning conversations around student work, similar themes and developments may emerge. It may be possible, then, to create a curriculum for leading workgroups for other professional educators focused on helping teachers begin the process of examining student work.

2 During this first year, for practical and pedagogical reasons, the facilitators chose which problems to pose next. We wanted to have the chance to share what we observed and make decisions about what kind of problem would help us move the group's thinking along. In the second year, while we were still facilitating the workgroups, teachers were more involved in deciding what questions to try. In subsequent years, the teachers took over facilitation of the workgroups, and they decided which tasks to try with their students.

3 (W1: 9/30/97). This notation is used throughout the text to reference the data source. "W" followed by a number indicates the workgroup in which the exchange occurred. The date of the workgroup follows.

4 The term invented algorithm is used in CGI frameworks. It refers to student-generated methods that are efficient and typically generalize beyond the specific numbers being used.

5 Facilitator B designates the contribution of the second author who attended this last meeting.

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