Exploring Colleagues' Professional Influence on Mathematics Teachers' Learning

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Background/Context: *This article contributes to the literature on how teachers learn on the job and how schools and districts can support teacher learning to improve student learning and incorporate changing standards and curricular materials into instructional practices. The findings in this study are relevant to the implementation of ambitious mathematics instruction reform through changing teachers' knowledge and instructional practices.*

Focus of Study: *This study examines how middle school teachers' networks influence their mathematical knowledge for teaching (MKT) and instructional practices. We also examined how mathematics coaches' expertise, in the form of MKT, plays a role in augmenting the extent to which teachers learn through interacting with close colleagues.*

Research Design: *The article draws on data from a larger NSF-funded study in four large, urban districts that responded to accountability pressures by attempting to implement ambitious mathematics instruction aligned with the recommendations of the National Council of Teachers of Mathematics (NCTM) and by supporting implementation with a significant investment in teacher learning. The analysis in this paper involves 89 focal participants who were middle school mathematics teachers in 29 schools, the focal participants' close colleagues, and their instructional coaches. Measures include mathematics teachers' professional networks, MKT, classroom practices, individual background characteristics, and school factors. We used hierarchical linear models with cross-level interaction effects and in-depth sensitivity analyses of the effects of close colleagues and coaches.*

Findings/Results: *Our results show that changes in teachers' instructional practice were positively related to their access to instructional expertise through interactions with close colleagues. But, we did not find a similar significant relationship between changes in teachers' MKT and access to their close colleagues' MKT expertise. Rather, coaches' MKT expertise positively moderated the extent to which teachers learned MKT from their close colleagues through seeking advice*

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on teaching mathematics; that is, having an expert coach in the school enhanced the MKT learning opportunities that teachers had from interacting with close colleagues.

Conclusions/Recommendations: *Results from this study shed light on how to support teachers' on-the-job learning and successfully implement ambitious instructional reforms in schools. It is important for schools and districts to consider ways to encourage the development of teacher networks that can promote instructional changes. For example, schools and districts can purposely provide common planning time and common workspaces that facilitate sharing expertise among teachers. They can also support teachers with instructional coaches who have content expertise and know how to facilitate interactions among teachers.*

INTRODUCTION

The study of how teachers learn on the job has been a recurring and evolving topic in the literature as teachers work to improve student learning and incorporate changing standards and curricular materials into their instructional practices (e.g., Cobb & Jackson, 2011). This study aims to add empirical evidence to such literature by examining how teachers learn knowledge and practice from close colleagues on the job. It further examines how instructional coaching might play a special role with regard to teachers learning from close colleagues.

To support mathematics teachers' development of high-quality instructional practices, rather than merely "teaching to the test," the National Council of Teachers of Mathematics (NCTM, 2000) drew on available research to articulate a broad vision of instruction, often referred to as "ambitious teaching" (Cobb & Jackson, 2011; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). This vision aims to develop students' conceptual understanding of key mathematical ideas and procedural fluency, and their ability to communicate their mathematical reasoning effectively (Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000). To achieve these learning goals, teachers should (a) support students to solve cognitively demanding tasks (Stein, Smith, Henningsen, & Silver, 2000), (b) press students to provide evidence for their reasoning and to make connections between their own and their peers' solutions (McClain, 2002), (c) coordinate whole class discussions in which they build on students' contributions to achieve their mathematical goals for students' learning (Franke, Kazemi, & Battey, 2007; Stein, Engle, Smith, & Hughes, 2008), and (d) distribute learning opportunities equitably to all students in the classroom (Lampert & Graziani, 2009; NCTM, 2000). Teachers who attempt to teach in this way generally find it challenging to do so because instructional practices of this type contrast sharply with typical teaching in most U.S. classrooms and require teachers to anticipate and respond to students' thinking (Cobb & Jackson, 2011; Kazemi, Franke, & Lampert, 2009; Lampert et al., 2010).

To help teachers to develop the knowledge and practice for ambitious teaching, researchers, including those referenced above, have tried to understand and articulate aspects of this knowledge and practice in attempts to understand how to better support teachers to develop such practice (e.g., Shulman, 1986; Ball, Hill, & Bass, 2005; Grossman, 1995; Thompson, 1992). One thing that makes these attempts challenging is the interconnected nature of the knowledge and practice of teaching. Much knowledge is situated in the practice of teaching and, therefore, difficult to describe and develop outside of that context (Putnam & Borko, 1997). In addition, developing mathematical knowledge does not ensure high-quality instructional practices in classrooms (S. Cook & Brown, 1999). All of these reasons make it difficult for teachers to learn how to teach in ambitious ways and for schools and districts to effectively support the learning of such teaching approaches.

Using data collected from a large-scale, longitudinal project funded by National Science Foundation (NSF), this study aims to understand better how teachers develop the knowledge and practices necessary for ambitious instruction on the job. The larger project worked with four large, urban districts that were responding to accountability pressures by implementing ambitious mathematics instruction aligned with the recommendations of the NCTM (2000) and by supporting implementation with a significant investment in teacher learning. These supports for teacher learning included allocation of common planning time to support teacher collaboration, provision of content-focused instructional coaches, and provision of content-focused professional development at the district level. Investigating teacher learning occurring in these districts has the potential to provide useful lessons for other districts working to improve mathematics instruction. Specifically, in this study we set out to explore two questions: (a) Is access to expertise through interactions with close colleagues related to teachers' learning of mathematical knowledge for teaching and instructional practice? (b) Does the expertise of the coach influence the degree to which teachers learn mathematical knowledge for teaching and instructional practice through interactions with close colleagues?

In what follows in this paper, we describe the existing literature pertaining to mathematics teachers' knowledge and practice and teachers' on-the-job learning through interactions with close colleagues. We then describe our sample and methods that we used to explore our research questions. Briefly, our results show that changes in teachers' instructional practice were positively related to their access to instructional expertise through interactions with close colleagues. Moreover, coaches as content experts positively moderated the extent to which teachers learned mathematical knowledge for teaching from their close colleagues.

MATHEMATICAL KNOWLEDGE FOR TEACHING AND INSTRUCTIONAL PRACTICE

To facilitate teachers' learning of teaching mathematics, researchers have been investigating how to identify and measure the knowledge implicated in teaching mathematics for quite some time. Particularly, the recent work of Hill and colleagues attempts to account for the nature of mathematical knowledge needed specifically by teachers, which they call *mathematical knowledge for teaching (MKT;* Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2007*).* Hill and colleagues described mathematical knowledge for teaching as "multifaceted, including not only teachers' ability to solve the problems that their students are expected to solve, but also to understand the content in the particular ways needed for teaching it, to understand what students are likely to make of the content, and to craft instruction that takes into account both students and the mathematics" (Hill et al., 2007, p. 125). They created a pencil and paper assessment of this situated knowledge by building on observational and interview data (Hill et al., 2004). Items attempt to assess mathematical knowledge that is situated in the work of teaching. For example, an item might ask the test-taker to choose the most likely reason for a student's incorrect response based on a sample of that hypothetical student's work. Given that teachers often have to look at student work and attempt to understand their students' thinking, this item is intended to be closely related to the practice of teaching mathematics. Hill, Ball, and their colleagues have gone beyond theoretical connections between mathematical knowledge for teaching and mathematics teachers' instructional practice to empirically demonstrate those relationships. They found positive relationships between teachers' mathematical knowledge for teaching and student achievement (Hill, Rowan, & Ball, 2005; Hill Ball, Blunck, Goffney, & Rowan, 2007; Hill et al., 2008).

While teachers' instructional practices are easier to document than their knowledge, the question of how to measure those practices is challenging and persistent. Researchers have attempted a variety of different approaches to measuring instructional practice, including teacher surveys, logs, and observations. Researchers have used teacher surveys or logs to minimize costs, but this data comes with the validity problems inherent to self-reported data (Wubbels, Brekelmans, & Hooymaters, 1992). Therefore, observations are the most common approach for measuring teachers' instructional practice. Yet, observational measures of instructional practice are not necessarily straightforward ways to assess instructional practice. Despite the fact that instructional practice is observable, objective observer assessment requires careful decisions about how and what to observe, which may privilege some teacher actions and ignore others (Volpe, DiPerna, & Hintze, 2005). Some observational instruments are content-neutral (e.g., *Classroom Assessment Scoring System,* or CLASS), while others focus specifically on assessing the teaching of a particular subject area (e.g., *Mathematical Quality of Instruction,* or MQI, and *Instructional Quality Assessment,* or IQA). Even within a particular subject area (e.g., mathematics), there is considerable variation with regard to what might be measured by a particular set of rubrics. For example, the MQI places emphasis on the *mathematical* quality of instruction (Hill et al., 2008), while the IQA places more emphasis on level of challenge of the mathematical activities within the classroom (Boston & Wolf, 2006). We chose to use the IQA instrument because it was well aligned with the four districts' goals for mathematics instruction. In particular, their goals included a launch-explore-summarize lesson structure in middle school mathematics classrooms, which the IQA assumed.

LEARNING MATHEMATICAL KNOWLEDGE FOR TEACHING AND AMBITIOUS TEACHING PRACTICE FROM CLOSE COLLEAGUES AND COACHES

Professional development programs have been used by most districts to provide teachers with on-the-job opportunities to learn, and teacher interactions often manifest characteristics of effective professional development, such as (a) involves active learning, (b) grounded in teachers' practice, (c) coherent with other learning opportunities, (d) focused on content, (e) involves collective participation of teachers from the same school or grade, and (f) ongoing in duration (Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Horn & Little, 2010; Putnam & Borko, 2000; Sun, Penuel, Frank, Gallagher, & Youngs, 2013; Wilson & Berne, 1999). On-the-job interactions with colleagues have the potential to have many of these characteristics and, hence, are likely to provide opportunities for teacher learning. For example, if a teacher talks to her colleagues about how students might solve a particular task within an upcoming lesson, this involves active learning, is grounded in her teaching practice, is focused on content, involves collective participation amongst colleagues, and has the potential to be both coherent with other learning opportunities and ongoing in duration (in the case that it is part of an ongoing dialogue).

Besides formal professional development sessions, interactions among teachers often happen in informal settings through seeking advice from each other, such as spontaneous conversations about students' learning, sharing teaching materials, and observing each other's teaching. Since the knowledge and practice we intended to measure in this study pertains to classroom teaching, the colleagues with whom a teacher works closely and shares similar working contexts become immediate resources that the teacher can draw on for help with instructional matters (Putnam & Borko, 2000). When interactions involve activities that give rise to deep, critical reflection on practice, colleagues' knowledge and instructional expertise can be a major source of professional growth for teachers (Bidwell & Yasumoto, 1999; Bryk & Schneider, 2002; Horn, 2010; Horn & Little, 2010; McLaughlin, 2006). In such activities, teachers benefit from exposure to information that is embedded in classroom practices that colleagues can make explicit, especially when those colleagues possess relevant instructional expertise and local knowledge (e.g., Darling-Hammond & McLaughlin, 1995; Webster-Wright, 2009). This expertise can be shared when teachers interact and collaborate with each other to address commonly identified classroom problems (Penuel, Riel, Krause, & Frank, 2009). Moreover, regular interactions among teachers can also bring about coherent instructional practices among teachers schoolwide by developing and communicating common educational goals and pedagogical strategies (e.g., Coburn, 2001; Park & Datnow , 2009; Penuel et al., 2009; Penuel, Frank, Sun, Kim, & Singleton, 2013; Sargent & Hannum, 2009).

To support teachers' development of ambitious teaching practices, it has also become increasingly common for districts to employ content-focused instructional coaches. Coaches can support teachers in developing a range of knowledge, notably including mathematical knowledge for teaching, district recommendations about curriculum implementation and pacing, and how to use results from the state-standardized tests to gain insight on student learning, etc. (e.g., Coburn & Woulfin, 2012; Feldman & Tung, 2002). They can also affect individual teachers' development by influencing how and about what teachers interact with each other. Coaches can change the "routines of interaction" to in-depth discussions of mathematical ideas and how students think mathematically, rather than merely sharing materials and discussing the context of instruction (Coburn & Russell, 2008, p.218). Coburn and Russell (2008) found that teachers who had access to coaches with greater expertise and skill in establishing effective routines of interaction among teachers had conversations with other teachers that were more likely to involve pedagogical principles, the nature of mathematics, or how students learn, when compared with those who did not have the same access to such coaches. Further, those teachers who were effectively coached were more likely to have such conversations even when the coach was not around.

However, overall, prior studies have provided limited empirical evidence regarding how different aspects of knowledge or practice are learned through teachers' interactions with colleagues and how coaches' expertise may influence the learning of knowledge or practice through these interactions with colleagues. In this study, we hypothesize that advice-seeking interactions with colleagues have the potential to support the development of teachers' knowledge and practice. Within conversations with colleagues, teachers might describe how they teach particular mathematical concepts, why some concepts are difficult for students, and how they react to certain students' strategies or difficulties. Thus, topics that might develop a teacher's mathematical knowledge for teaching could arise in conversations among colleagues. Similarly, teachers might describe different instructional strategies or ways that they teach particular ideas, which could influence the instructional practices of their colleagues. Lastly, mathematics coaches who are experts might stimulate conversations among teachers, provide better language for teachers to articulate various aspects of their knowledge and practice, or establish routines of interaction among teachers around mathematics instruction, thereby enhancing the extent to which teachers can learn from colleagues.

METHODS

SAMPLE

To investigate our research questions regarding teachers' learning of knowledge and practice, this study draws on data from the Middle-school Mathematics and the Institutional Setting of Teaching (MIST) project, an ongoing NSF-funded project that uses a design-based approach to investigate school and district supports for middle school mathematics teachers' development of ambitious and equitable instructional practices on a large scale (Cobb & Jackson, 2011; Cobb & Smith, 2008). The MIST project collected data from four large urban districts. In each of the four districts, approximately 30 teachers and their instructional leaders (principals, assistant principals, and coaches) participated in the study. For the purpose of this study, we used data collected in the 2008–2009 and the 2009–2010 school years on mathematics teachers' (a) professional networks, (b) mathematical knowledge for teaching, (c) classroom practices of teaching mathematics, and (d) teacher background characteristics and school organizational and contextual factors (e.g., principal leadership, school culture, curriculum alignment, and the number of hours participating in mathematics professional development).

Among 108 teachers who were full teacher participants in the MIST project in both the 2008–2009 and the 2009–2010 school years, 89 teachers from 29 schools were involved in the final analysis because 19 teachers had missing data on one of the key variables in the final models.[1] As shown in Table 1, these teachers had an average of 11.32 years of mathematics teaching experience as of the 2009–2010 school year. About 74% of teachers were female; about 64% were Caucasian. About 92% of teachers were certified to teach either middle or secondary grades.

Table 1. Teacher Background Characteristics in 2009–2010

Variables	Mean of Final Sample	Mean of the Full Sample in 2009–2010
Years taught mathematics	11.32 (9.37)	9.91 (8.83)
Percentage of teachers who were female	73.86%	68.46%
Percentage of teachers who were Caucasian	63.64%	64.89%
Percentage of teachers who were certified to teach either middle or secondary grades	92.05%	86.36%

Notes. These percentages of the final sample of 89 teachers are not significantly different from the results based on the whole sample of 108 teachers who were full participants in 2009–2010. This reduces the concern that deleting missing data in the final analysis would lead to a biased sample. Standard deviations are included in the parentheses.

MEASURES

Dependent Variables

Consistent with our theoretical framing, we primarily investigated two types of teachers' knowledge and practice: *mathematical knowledge for teaching (MKT)* and *instructional practice*. In this section, we introduce our dependent variables and focal independent variables. Other independent variables that were included in the models but are not central to this analysis are explained in detail in Appendix A.

MKT in 2009–2010: This measure captures teachers' content knowledge that is relevant to teaching middle school mathematics, which was indicated by teachers' 2010 composite scores on the Mathematical Knowledge for Teaching Assessment. This instrument was developed by the *Study of Instructional Improvement/Learning Mathematics for Teaching* (Hill et al., 2004). The instrument has a reliability index of 0.70 or above and can be used to assess teachers' knowledge growth with respect to two dimensions: number concepts and operations (NCOP); and patterns, functions, and algebra (PFA). For each of the two subtests (NCOP and PFA), raw scores were translated into IRT (item response theory) scale scores (provided by MKT developers), the determination of which was based on results from a pilot administration of the assessment to a national sample of approximately 640 practicing middle school teachers. For our analyses, we used a combined average of these two scale scores to form a single MKT score for each participant in each year. The use of IRT scores based on the national sample allows us to interpret the MKT scores of the teachers in our sample in relation to the national average and distribution (i.e., a mean score of 0 and standard deviation of 1).

Instructional practice in 2009–2010: To collect data on teachers' instructional practice, videographers recorded two days of instruction in each participating teachers' classroom (consecutively, when possible, to account for the fact that a lesson might extend over more than one day) in January, February, or March of each year. We asked teachers to include a problem-solving activity and a related whole-class discussion in their instruction. The goal of the video recordings was not to capture the nature of teachers' everyday practice, but rather to assess the quality and extent to which a teacher might enact the particular kind of instruction articulated by district leaders as the goal of the instructional reform. Given our request that teachers include a problem-solving lesson and a whole-class discussion, it would be appropriate to think of what was video recorded as teachers' best shot at enacting ambitious instructional practices.

The two video-recorded lessons for each teacher were coded using the *Instructional Quality Assessment* (IQA; Boston & Wolf, 2006). The IQA, developed at the University of Pittsburgh, is based on the Mathematical Tasks Framework (Stein, Grover, & Henningsen, 1996), and is consistent with the districts' ambitious instructional visions and professional development programs. The instrument is designed to measure the cognitive demand, or level of challenge, of the task as it appears in curricular materials, the cognitive demand of the task as implemented, and the quality of the concluding whole-class discussion (or "summarize" portion of the lesson). The IQA rubrics have been determined to be sufficiently reliable and valid by the IQA developers (Boston & Wolf, 2006; Matsumura, Garnier, Slater, & Boston, 2008). In our use of the IQA rubrics, we required 80% rater agreement during training and maintained 75.4% exact agreement (Cohen's kappa of 0.48) throughout the coding process.

Teachers' instructional practice was scored separately for each lesson on eight rubrics with a scale that ranges from 0 to 4. Two of these eight rubrics deal with the cognitive demand of the task, and the other six pertain to the quality of whole-class discussion. Since the purpose of the video recordings of two consecutive days of instruction was to capture teachers' best shot at high-quality math instruction, we used the recording with the higher set of scores to assess the quality of teachers' instructional practice. The observed scores on the eight rubrics for the best day were then aggregated to a composite score by using graded response model[2] (Muraki & Bock, 1991) to produce a psychometrically sound measure of classroom practices for each teacher.

Focal Independent Variables

Access to close colleagues' MKT: To capture the effects of teachers' interactions with close colleagues on teacher learning, on our teacher survey in 2009–2010, we asked teachers to list colleagues they had turned to for advice or information about teaching mathematics in the last school year. To approximate teachers' access to close colleagues' MKT through interactions, we followed the approach of our prior work (Frank, Zhao, & Borman, 2004) and defined exposure as a function of the frequency of interactions (0= "no interaction," 1= "a few times per year," 2= "once or twice per month," 3= "once or twice per week," and 4= "daily or almost daily"[3]) and potential knowledge available through the interaction (approximated by the MKT in the prior, 2008–2009, school year). For example, if Lisa interacted with Bob with a frequency of twice per month during the whole school year (frequency of 2), and Bob had a prior MKT level of 1.02 in 2008–2009, then Lisa's access to MKT (via Bob) is 2 x 1.02= 2.04. To combine information across a teacher's network, we took the sum across all teachers that teacher *i* identified as colleagues from whom they sought advice:

$$\text{Access to close colleagues' } MKT_i = \sum_{\substack{i'=1, \\ i' \neq i}}^{n_{i'}} (\text{the frequency of interaction}_{ii'}) \times (\text{help provider's prior } MKT_{i'}) \quad (1)$$

Where in equation (1), $n_{i'}$ is the number of teachers that person *i* indicated as providing advice about mathematics instruction, the *frequency of interaction*$_{ii}$ represents the frequency of interactions between teacher *i* (e.g., Lisa) and teacher *i'* (e.g., Bob). The *help provider's prior MKT*$_{i'}$ represents the prior MKT score in 2008–2009 for teacher *i'* (e.g., Bob).

Access to close colleagues' instructional practice: To derive a proxy of the access to close colleagues' mathematics instructional practice, we used a procedure similar to the one demonstrated in equation (1), except that we replaced *help provider's prior MKT* $_{i'}$ with *help provider's prior IQA score* $_{i'}$.

School level variable: Coach's expertise in 2009–2010: For both outcomes, we used coaches' mathematical knowledge for teaching in 2009–2010 to approximate their expertise. We were unable to document coaches' instructional practices as a form of expertise due to the fact that many of the coaches in our sample were full-time coaches who did not currently teach. If there was more than one coach working with the school, we chose the coach who had a higher level of MKT to capture the maximum possible expertise that a teacher could access via a coach in the school.

ANALYTIC STRATEGIES

Given the nested structure of the data (teachers nested within schools), we used hierarchical linear models (HLM; see Raudenbush & Bryk, 2002) with two levels: teacher level (level 1) and school level (level 2). We estimated the model separately for these two dependent variables: MKT and instructional practice. We first estimated unconditional models to examine the distribution of variance between the teacher level and the school level, and the results are included in Table 2. For each of these two dependent variables, there was a significant amount of variance allocated at the school level (p-value≤0.001), which supports the use of multilevel models to fit the data. Interestingly, we noticed that there was a larger percentage of school-level variance for teachers' MKT (35.28%) than for instructional practice (23.51%). That is, the distribution of teachers across schools varied more widely based on knowledge rather than instructional practice. In contrast, instructional practice among teachers varied substantially within schools.

Table 2. Variance Components

	MKT in 2009–2010	Instructional practice in 2009–2010
Teacher-level variance component	0.446	0.916
School-level variance component	0.243***	0.282***
The percentage of school-level variance out of total variance	35.28%	23.51%

Note. *p-value≤0.1; **p-value≤0.05; ***p-value≤0.001

The models we employ aim to understand how change in either teachers' MKT or teachers' instructional practice was a function of access to close colleagues' expertise, while controlling for several aspects of the institutional setting and other potential learning opportunities. We also examine whether coach expertise moderates the change as a function of access to close colleagues' expertise. However, potential challenges to causal inference based on observational data demand strategic approaches to eliminate alternative explanations for the development of teachers' expertise, as measured by both knowledge and practice. First, to statistically account for variation in prior individual differences that may confound with teachers' knowledge and practice, we controlled for the teachers' prior knowledge and practice, we controlled for the prior teachers' knowledge and practice in the 2008–2009 school year. Recently, T. Cook, Shadish and Wong (2008) and Shadish, Clark, and Steiner (2008) showed that estimates from nonrandomized studies that included controls for the precondition of the outcome variable closely approximated estimates from randomized experiments. Controlling for the prior substantially reduces the likelihood of prediction errors and, thus, increases the precision of estimation. Furthermore, the prior scores "absorb" the influence of other unmeasured and sustaining characteristics of teachers, such as personal values and motivation to collaborate with other teachers (Frank, 2000). Moreover, controlling for prior knowledge or practice reduces the potential of residuals' non-normality and, therefore, increases the consistency of estimates (Raykov & Marcoulides, 2008).

In addition to adjusting for individual teachers' prior knowledge or practice, we also accounted for other variables that may confound the relationship between exposure to close colleagues' expertise and changes in the two outcomes of interests, including other types of school and district supports for teacher learning (e.g., principals' instructional leadership as approximated by *principals' expectations on high-quality mathematics instruction in 2009-10*, professional development for teaching mathematics as approximated by *professional development hours in 2009-10*, and *teachers' perception of curriculum alignment;* detailed measures of these variables are included in Appendix A), and individual teachers' background information (e.g., *Years taught mathematics up to 2009-10, Being a Caucasian, and Being certified to teach either middle or secondary grades in 2009-10;* see Appendix A for detailed measures). These variables were included in the models to explain some variance in teachers' knowledge and practice, rather than explain the variation in teachers' interactions.

At the school level, we controlled for school means of knowledge and practice, in models with outcomes of knowledge and practice, respectively, to account for school norms. We also used district fixed effects (including district dummy variables, coded as District A, District B, District D, using District C as the reference[4]) to account for each district's unique background characteristics. We grand-mean centered all continuous independent variables in models to make the coefficients easy to interpret. The resulting models can be simplified as follows:

Level-1 Model, Teacher Level

MKT (or Instructional practice) in 2009-10$_{ij}$= β_{0j}
+ β_{1j} *Access to close colleagues' prior MKT (or instructional practice)* $_{ij}$
+ β_{2j} *Professional development hours in 2009-10* $_{ij}$
+ β_{3j} *Perceived curriculum alignment in 2009-10* $_{ij}$
+ β_{4j} *Principals' expectations on high-quality mathematics instruction in 2009-10*$_{ij}$
+ β_{5j} *Prior MKT (or instructional practice) in 2008-09*$_{ij}$ (2)

$+ \beta_{6j}$ *Years taught mathematics up to 2009-10* $_{ij}$
$+ \beta_{7j}$ *Being certified to teach either middle or secondary grades in 2009-10* $_{ij}$
$+ \beta_{8j}$ *Being a Caucasian* $_{ij} + e_{ij}$

In equation (2), *MKT (or Instructional practices) in 2010*$_{ij}$ indicates the MKT or instructional practice of teacher *i* in school *j* in 2009–2010. β_{0j} is the level-1 intercept. β_{qj} (q=1, 2, 3, 4, 5, 6, 7, 8) represents the coefficient estimate corresponding to each of the predictors. e_{ij} is the level-one residual, assumed normally distributed with mean 0 and variance of σ^2.

Level-2 Model, School Level

$\beta_{0j} = \gamma_{00} + \gamma_{01}$ *Coach's expertise in the school in 2009-10*$_j$
$+ \gamma_{02}$ *School mean of prior MKT or instructional practice in 2008-09*$_j$
$+ \gamma_{03}$ *Being in District A*$_j$ (3)
$+ \gamma_{04}$ *Being in District B*$_j$
$+ \gamma_{05}$ *Being in District D*$_j$ $+ u_{0j}$

In equation (3) γ_{00} is the level-2 intercept. u_{0j} is the level-2 residual, which is assumed to be normally distributed with mean zero and variance τ_β. We used γ_{01}- γ_{05} to test fixed effects of corresponding predictors. Other level-1 slopes were fixed at level-2 with intercepts only. To examine the extent to which coaches' expertise would moderate learning from close colleagues, we included a cross-level interaction term between a *coach's expertise* (at school level) and the *access to close colleagues' prior MKT or instructional practice* $_{ij}$ (at the teacher level). β_{1j} in equation (3) was then modeled as follows:

$\beta_{1j} = \gamma_{10} + \gamma_{11}$ *Coach's expertise in 2009-10*$_j$ (4)

In equation (4), γ_{11} indicates the extent to which a coach's expertise would change teachers' MKT or practice through influencing their learning from close colleagues.

In addition, to rule out alternative explanations for the influences of close colleagues and coaches on teachers' learning in our sample and to better understand our findings, we conducted several in-depth sensitivity analyses. We separated close colleagues into those who were coaches vs. those who were regular teachers, to examine whether a particular group of close colleagues were more likely to influence teacher learning. Moreover, to further understand the coaching effect, we conducted analyses on different coaching practices in these four districts, by (a) separating coaches into two groups (district coaches vs. school-based coaches); (b) comparing the effects of the dummy variable of whether a coach was in the school, the coach's expertise, and the frequency of coaching; and (c) contrasting the coaching practices in two districts that had different gains in MKT—District B and district D.

RESULTS

Table 3 includes descriptive statistics of key variables in the models, and Table 4 includes correlation coefficients among key level-1 variables after partialling out the prior measures of MKT and instructional practice in 2008–2009 in the models. We then present the estimated results from HLM models for MKT in Table 5 and results for IQA in Table 6. Model-I in Table 5 and Table 6 includes only main effects, while Model-II includes both main effects and cross-level interaction effects (i.e., examines the moderating effect of coach expertise on learning through interactions with close colleagues).

RESULTS FROM ESTIMATING MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

As indicated in Table 3, the average level of teachers' MKT in 2009–2010 was -0.056, which was an increase from the average of -0.148 in 2008–2009, and the gain=0.092, which is on the border of statistical significance level of 0.05 (t-value of paired t test =1.91, p-value=0.0596). As shown in Table 4, after accounting for the prior MKT, teachers' MKT in 2009–2010 has a close to zero correlation with the access to close colleagues' prior MKT through interactions.

Table 3. Descriptive Statistics of Key Variables in HLM Models

Variables	Mean
Teacher-Level Variables (N=89)	
MKT in 2009-10	-0.06 (0.81)
MKT in 2008-09	-0.15 (0.82)
Access to close colleagues' prior MKT	3.74 (5.21)
Instructional practice in 2009-10	0.05 (1.08)
Instructional practice in 2008-09	-0.004 (0.98)
Access to close colleagues' prior instructional practice	3.70 (6.09)
Professional development hours in 2009-10	28.93 (12.35)
Perceived curriculum alignment in 2008-09	3.04 (0.63)
Principals' expectations on mathematics instruction in 2009-10	2.33 (0.68)

Variables	Mean
Years taught mathematics up to 2009-10	11.26 (9.37)
Being certified to teach either middle or secondary grades in 2009-10	0.91 (0.29)
Being a Caucasian	0.64 (0.48)
School-Level Variables (N=29)	
Coach's expertise in 2009-10	2.16 (1.71)
School mean of prior MKT in 2008-09	3.94 (0.49)
School mean of prior instructional practice in 2008-09	2.12 (0.32)

Note. Standard deviations are included in the parentheses.

Table 4. Partial Correlation Coefficients among Teacher-Level Key Variables

	1	2	3	4	5	6	7	8	9	10
1. MKT in 2009-10	1.000									
2. Access to close colleagues' prior MKT	0.045	1.00								
3. Instructional practice in 2009-10	0.136	0.102	1.000							
4. Access to close colleagues' prior instructional practice	-0.063	0.867***	0.098	1.000						
5. Professional development hours in 2009-10	0.179*	-0.009	-0.061	-0.081	1.000					
6. Perceived curriculum alignment in 2009-10	0.251**	0.132	-0.074	0.103	0.159	1.000				

	1	2	3	4	5	6	7	8	9	10
7. Principals' expectations on mathematics instruction in 2009-10	0.158	0.014	0.094	-0.011	0.138	0.184*	1.000			
8. Years taught mathematics in 2009-10	-0.001	-0.122	-0.241**	-0.124	0.077	0.185*	-0.095	1.000		
9. Being certified to teach either middle or secondary grades in 2009-10	0.018	-0.001	-0.034	0.016	-0.19*	-0.097	0.018	-0.238**	1.000	
10. Being a Caucasian	0.074	0.019	0.094	0.045	0.119	-0.008	-0.217*	0.108	-0.101	1.000

Note. *p-value≤0.1; **p-value≤0.05; ***p-value≤0.001; The correlation coefficients were estimated after partialling out priors (i.e., prior MKT or prior instructional practice).

These results are generally consistent with HLM results from Model-I in Table 5. After controlling for other covariates, the main effect of access to close colleagues' prior mathematical knowledge for teaching through seeking advice is not statistically significant. At the school level, the estimate of the main effect of coach's expertise on teachers' MKT was not statistically significant either. However, in Model-II, the estimate of the cross-level interaction term between coach's expertise (at school level) and access to close colleagues' prior mathematical knowledge for teaching through seeking advice (at the teacher level) is positively significant at the 0.05 significance level (unstandardized coefficient β=0.012, standardized coefficient b=0.182). If a school had a math coach with high MKT, teachers in this school would be significantly more likely to learn MKT from their close colleagues through professional interactions. That is, coach expertise has an indirect effect on the gain in teachers' mathematical knowledge, in effect, enhancing learning from close colleagues.

Table 5. Results of Estimating Mathematical Knowledge for Teaching (MKT)

	Model-I		**Model-II**	
	Unst.d[(a)]	**St.d[(b)]**	**Uns.d[(a)]**	**St.d[(b)]**
Teacher level (N=88)				
Access to close colleagues' prior MKT	0.005 (0.012)	0.045	-0.028 (0.021)	-0.045
Professional development hours in 2009-10	0.006 (0.004)	0.098	0.005 (0.004)	0.082
Perceived curriculum alignment in 2009-10	0.157** (0.079)	0.135**	0.149* (0.078)	0.128*
Principals' expectations on high-quality mathematics instruction in 2009-10	0.079 (0.062)	0.078	0.064 (0.061)	0.062
Prior mathematical knowledge for teaching (MKT) in 2008-09	0.726*** (0.063)	0.726	0.754*** (0.064)	0.755
Years taught mathematics up to 2009-10	-0.007 (0.005)	-0.081	-0.008 (0.005)	-0.091
Being certified to teach either middle or secondary grades in 2009-10	0.202 (0.184)	0.107	0.241 (0.182)	0.128
Being a Caucasian	0.028 (0.117)	0.018	0.065 (0.117)	0.041
School Level (N=29)				
Coach's expertise in 2009-10	0.017 (0.06)	0.038	0.035 (0.059)	0.058
School mean of prior MKT in 2008-09	0.303* (0.17)	0.605*	0.304* (0.166)	0.607*
Being in District A	0.164 (0.247)	0.095	0.16 (0.241)	0.092
Being in District B	-0.201 (0.186)	-0.124	-0.215 (0.181)	-0.133
Being in District D	-0.114 (0.232)	-0.058	-0.185 (0.229)	-0.094
Cross-level Interactions				
Access to close colleagues' prior MKT × Coach's expertise in 2009-10			0.012** (0.006)	0.182**

Note. *p-value≤0.1; **p-value≤0.05; ***p-value≤0.01 Standard errors are included in parentheses. [(a)]Unst.d means unstandardized estimates, while [(b)]St.d means standardized estimates.

RESULTS FROM ESTIMATING INSTRUCTIONAL PRACTICE

The average quality of teachers' instructional practice increased from-0.004 in 2008–2009 to 0.046 in 2009–2010 as indicated in Table 3. The average gain of 0.05 is not statistically significant (t-value of paired t test=0.39, p-value=0.7). The variation in instructional practice among teachers in 2009–2010, after partialling out their prior instructional practice, is positively related to the variation in the access to their colleagues' prior instructional practice through seeking advice (see Table 4, correlation coefficient ρ=0.098). Further, by controlling for other covariates in the model, as indicated by Model-I in Table 6, at the teacher level, the direct exposure to close colleagues' prior instructional practice had a significant positive effect on the improvement of teachers' own instructional practices at 0.1 significance level (β= 0.042, b = 0.276, p-value=0.075). Although the p-value of this estimate is slightly larger than the conventional 0.05 significance level, due to our relatively small sample size and, thus, fewer degrees of freedom and the fact that its estimate is the single largest positive effect among all predictors at the individual teacher level, we posit that this effect of exposure to close colleagues is substantial in practice. Specifically, one standard deviation increase in exposure to close colleagues' prior instructional practice would lead to an increase of 0.276 standard deviations in instructional practice in 2009–2010 for a typical teacher in the sample.

At the school level, Table 6 Model-I shows that the main effect of mathematics coaches' expertise on teachers' instructional practice was not statistically significant. The cross-level interaction term between coach's expertise and teachers' access to close colleagues' prior instructional practice through interactions in Model-II was not statistically significant either, though still positive.

Table 6. Results From Estimating Mathematics Instructional Practice

	Model-I		Model-II	
	Unst.d[(a)]	**St.d[(b)]**	**Uns.d[(a)]**	**St.d[(b)]**
Teacher level (N=89)				
Access to close colleagues' prior instructional practice	0.042* (0.024)	0.276*	0.04 (0.025)	0.221
Professional development hours in 2009-10	0.007 (0.008)	0.086	0.007 (0.009)	0.08
Perceived curriculum alignment in 2009-10	-0.557*** (0.176)	-0.378***	-0.569*** (0.178)	-0.386***
Principals' expectations on high-quality mathematics instruction in 2009-10	0.349** (0.163)	0.269**	0.329** (0.166)	0.253**
Prior instructional practice in 2008-09	0.19* (0.113)	0.19*	0.211* (0.117)	0.21*

	Model-I		Model-II	
	Unst.d[(a)]	**St.d[(b)]**	**Uns.d[(a)]**	**St.d[(b)]**
Years taught mathematics up to 2009-10	-0.026** (0.011)	-0.233**	-0.028** (0.011)	-0.245**
Being certified to teach either middle or secondary grades in 2009-10	0.054 (0.359)	0.023	0.051 (0.36)	0.021
Being a Caucasian	-0.487* (0.273)	-0.241*	-0.463* (0.275)	-0.23*
School Level (N=29)				
Coach's expertise in 2009-10	-0.146 (0.108)	-0.261	-0.142 (0.109)	-0.254
School mean of prior instructional practice in 2008-09	1.53*** (0.505)	0.518***	1.496*** (0.51)	0.507***
Being in District A	0.131 (0.463)	0.06	0.133 (0.465)	0.06
Being in District B	-0.458 (0.376)	-0.222	-0.473 (0.379)	-0.229
Being in District D	1.496*** (0.435)	0.598***	1.476*** (0.438)	0.59***
Cross-level Interactions				
Access to close colleagues' prior instructional practice × Coach's expertise in 2009-10			0.009 (0.013)	0.094

Notes: *p-value≤0.1; **p-value≤0.05; ***p-value≤0.001 Standard errors are included in parentheses. [(a)]Unst.d means unstandardized estimates, while [(b)]St.d means standardized estimates.

IN-DEPTH ANALYSIS OF LEARNING FROM CLOSE COLLEAGUES AND INSTRUCTIONAL COACHING

Findings described above suggest that teachers in our sample were more likely to develop their instructional practices when they accessed instructional expertise of close colleagues, while they were more likely to learn MKT from close colleagues when they were in a school with a coach with more developed MKT. To better understand our findings and rule out alternative explanations, we conducted further analyses of the influences of close colleagues and coaches.

We were concerned about whether coaches who were also close colleagues in teachers' networks might have had a larger influence on teachers' learning MKT than other regular teachers as close colleagues. We thus constructed two measures of access to MKT, from colleagues who are teachers and colleagues who are coaches respectively, to examine whether, on average, teachers would learn more from coach colleagues than from other math teachers or vice versa. We did not find systematic differences in the effects of interactions with these two different types of close colleagues on learning MKT. We were not able to test for access to coaches' instructional practices through interactions because we did not collect data on coaches' instructional practices.

Moreover, while prior research suggests the expertise of the coach is critical (Coburn & Russell, 2008), we wanted to rule out other features of coaching practices as the potential reasons for our findings. First, we examined the effect of the different coaching models in the districts. There were two types of coaches working in sampled schools: district coaches and school-based coaches. We hypothesized that it was possible that school-based coaches might have more of an impact on the teachers' learning in the school they served than district coaches because school-based coaches were much more a part of the school's mathematics teaching and the school's faculty. We estimated models similar to those described in equations (3) and (4), but added a dummy variable for whether the school had a school-based coach. We did not find any significant difference in the impact of these two types of coaches.

Second, we examined whether the presence of coaches, the amount of time coaches spent in their schools, or coaches' expertise affected changes in teachers' knowledge and practice. To do so, we recoded the school-level coach variable in three ways (i.e., whether there was a coach in the school, number of hours per week the coach spent in the school, and the coach's MKT). By these three types of coding, we intended to find out whether simply having a coach in the school, the availability of the coach, or coaches' expertise matters more in augmenting learning from close colleagues. We found the variable of coach's expertise had the largest effect among these three coding schemes, although the other two variables had the same direction of effects (expertise *Cohen's d*=0.571; dummy coaching *Cohen's d*=0.487; frequency of coaching *Cohen's d*=0.317).

Third, to find further evidence that coaches' expertise matters, we conducted a comparison of coaches' expertise and frequency of working in schools across these four districts. We found two contrasting districts, District B and District D, in that these two districts had different gains in average MKT and different coaching expertise. While these two districts had similar district average of MKT in 2008–2009 to begin with (District B=-0.16, District D=-0.12), District B has a slightly negative gain in average MKT (gain=-0.01, the average MKT in 2009–2010=-0.17); in comparison, District D positively gained 0.08 units in average MKT from 2008–2009 to 2009–2010 (the average MKT in 2009–2010=-0.04). We further compared the coaching practices in these two districts. Both districts have coaches working frequently in schools (District B had coaches working in schools about 5 days per week, and District D coaches, on average, worked in schools 3.86 days per week).

However, the average coach expertise in District B as approximated by coaches' MKT in 2009–2010 (-0.39), was significantly lower than the average coach expertise in district D (0.96) (t-value =4.45, p-value≤0.01). This may further imply that the expertise of the mathematics coaches in District D might explain the higher than average MKT gains for teachers in District D.

DISCUSSION AND IMPLICATIONS

In this study using data from four large, urban school districts, we sought to examine whether middle school teachers learn aspects of mathematical knowledge for teaching and instructional practice from close colleagues and how mathematics coaches' expertise plays a role in enhancing teachers' learning opportunities. Results show that teachers made positive changes to their instructional practices when they had access to colleagues with more developed instructional practices. However, our data did not show such positive association between changes in MKT and access to close colleagues' expertise in MKT through interactions on teaching mathematics. But, coaches' expertise moderated the main effect of teachers' learning through access to close colleagues' expertise in MKT. In other words, coaches' expertise in MKT might augment the extent to which teachers can learn from interacting with close colleagues.

These findings are generally not surprising, given conventional wisdom about learning. We know that learning occurs through coparticipation in activities that are close to practice with individuals who are relatively accomplished (Bruner, 1996; Lave & Wenger, 1991). Advice-seeking interactions with colleagues theoretically have the potential to be rich sites for teacher learning. Further, these differences in teacher learning of MKT and practice are predictable when considering teachers' typical topics of conversation within interactions with colleagues. Some of the most common topics of conversation are the sharing of materials, activities, or instructional strategies. These topics are much more common than detailed discussions of particular mathematics problems—whether it is how they might be solved or how their students might solve them—which seem to be the most likely way for teachers to develop their MKT (Ball & Bass, 2002; Ball et al., 2005; Ball, Thames, & Phelps, 2008). Therefore, it is not surprising that interactions with close colleagues are more likely to develop teachers' instructional practice than their MKT. The findings from this study provide empirical evidence that teachers learn through their interactions with close colleagues and that there are important distinctions regarding which aspects of knowledge and practice teachers learn.

Further, the other key finding pertaining to the influence of the expertise of a mathematics coach within a teacher's school is consistent with existing literature. In particular, Coburn and Russell's (2008) finding that coaches influence teachers' routines of interaction with close colleagues resonates with our findings. The expertise of the coach is likely to influence whether and how coaches are able to influence the nature of teachers' interactions. In the case of our study, it is possible that coaches with more developed MKT are likely to focus teacher interactions toward detailed discussions of mathematics problems, which then provides opportunities for teachers to develop their MKT through interactions. Further, coaches pressing on interactions in this way might have a lasting impact on interactions by changing teachers' routines of interaction. In other words, teachers might continue to engage in detailed discussions of mathematics problems even when coaches are not present to facilitate the interactions in those ways. We believe that our finding extends that of Coburn and Russell because it provides quantitative evidence that coach expertise has an impact on teachers' learning opportunities through moderating collegial interactions on instructional matters.

Our findings have implications for districts and schools as they work to support teachers' learning of ambitious mathematics instruction. Our results suggest that teachers do learn through interactions with their colleagues. Hence, it is important for districts and schools to purposefully enact policies to encourage teacher interactions. Studies have shown that providing common planning time and designing common workspaces can promote teacher collaboration and further influence teachers' individual classroom practices (e.g., Flowers, Mertens, & Mullhall, 1999; Picucci, Brownson, Kahlert, & Sobel, 2002; Sargent & Hannum,2009). Strong principal instructional leadership that creates opportunities for teachers to interact around issues of instruction and student learning has been identified as another important factor to establish professional learning communities in schools (e.g., Blasé & Blasé, 2000; Fullan, 1991; Leithwood, Tomlinson, & Genge, 1996; Marks & Printy, 2003). Hence, while teachers' networks are emergent in nature, school and district policies can influence their development. Exactly which combination of factors and policies best supports the emergence of teacher networks is an area in which future research can further contribute.

Moreover, our findings suggest that the expertise of a teacher's close colleagues is integral to the extent to which that teacher can learn from collegial interactions. While not emphasized heavily, the inclusion of colleagues' expertise was a critical dimension of the way we defined access to expertise through interactions. One way to increase the amount of expertise in teacher networks is to provide content-focused instructional coaches who have expertise. To make content-focused coaching a viable support for teachers' development of knowledge and practice, districts and schools should consider the expertise of the coaches in hiring and in providing ongoing support and professional development for coaches. In addition, given our findings pertaining to an indirect effect of coach expertise through collegial interactions, it is likely that it is also important to consider the ways in which coaches engage teachers in collective and individual settings.

As in any empirical studies, the above discussion of our findings in this study is constrained by some data and analytical limitations. First, we include only 89 teachers in four urban school districts in the final analysis; therefore, findings from this study have limited generalizability to the population of urban middle schools in the United States with ambitious goals for mathematics instruction. We thus suggest replicating this study on a larger scale to verify the findings in this study. Second, although we attempted to control for alternative explanations of close colleagues' and coaches' influences on changing teachers' knowledge and practice, results in this study may still be subjected to other potentially confounding variables that were not controlled for in this study, such as the presence of concurrent school policy that may influence both teachers' gain in MKT (or gain in practices) and the change in teachers' network (or coaches' influences); thus, the estimation in this study is not causal.

Third, there are several limitations associated with our network measures. First, we use the frequency of interactions to approximate the variation in strength of teacher interactions (Frank et al., 2004). This measure does not imply the sustainability of teachers' interactions or the depth of interactions (see Coburn & Russell, 2008).[5] Second, most of our network data described within-school interactions. Our data do not allow us to model the differential influences on teachers' learning between within- and outside-school networks. These differences are worthy of investigation in future studies.

Finally, this study uses two measures of teachers' MKT and instructional practices to indicate the quality of their instruction and their possible effectiveness for promoting students' learning. In future studies, we plan to incorporate students' characteristics and student achievement data to produce value-added measures of teachers' effectiveness to further verify the results in this analysis.

CONCLUSION

This study provides quantitative evidence that teachers can learn from interactions with close colleagues and content-focused coaches. Our discussion in this study sheds light on the current practice of successfully implementing ambitious reforms on mathematics instruction by providing teachers with continuous on-the-job opportunities to learn new pedagogical content knowledge and classroom strategies for engaging students in cognitively demanding activities. Teachers can learn instructional practices through interacting with close colleagues' with instructional expertise, and learn content knowledge for teaching from instructional coaches who might augment the effects of learning from close colleagues.

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Notes

1. The exclusion of the 19 teachers from the analysis did not bias the results in HLM models. There were no statistical differences in teachers' MKT or on teachers' instructional practices between the 89 teachers who were included in the final sample and the 19 teachers who were excluded. Additionally, these 19 teachers were scattered across schools and no significant clustering within a particular school was found. Therefore, the 89 teachers in the final sample well represent the original sample.

2. Since the raters coded the IQA measures on a five-point scale according to the rubrics (0, 1, 2, 3, and 4), we chose item response models for more than two ordinal response categories to calibrate the levels of teachers' instructional quality. Among several models available in the literature, such as the graded response model (GRM), the partial credit model (PCM), and the rating scale model, we chose GRM to estimate the latent trait of instructional quality for several reasons. First, our theory predicts that teachers need to go through a series of steps to achieve a particular upper level category, which makes the rating scale and GRM models more persuasive than PCM because the latter does not assume the sequential relations between categories. Second, although all items were designed on a five-point scale, some of the items actually have only three or four categories that had observed values. Thus, the rating scale model that provides only one common set of category parameters for all items would not fit well to our data.

3. We considered recoding to days per year, but this exaggerated the most frequent behaviors, skewing the distribution of responses. The original survey scale used here is roughly the log of days per year.

4. We used District C as the reference district because this district has the lowest mean IQA and mean MKT in 2009–2010.

5. For instance, "sharing materials" was categorized as lower depth of interactions than "analyzing students' work together to see why they did not get it," or "discussing a new instructional activity" (Coburn& Russell, 2008).

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Appendix A

List of the Covariates

Prior MKT in 2008-09: This measure was derived similarly to *MKT in 2009-10.*

Prior instructional practice in 2008-09: This measure was derived similarly to the way of *instructional practice in 2009-10.*

Professional development hours in 2009-10: On the annual teacher survey, teachers were asked to report how many hours they had spent in professional development workshops or seminars in mathematics or mathematics education in the 2009-10 school year, including the 2009 summer, on a five rating scale: 0= "0 hours," 5="less than 6 hours," 10= "6-10 hours," 20= "16-35 hours," and 40= "more than 35 hours."

Perceived curriculum alignment in 2009-10: In our 2009-10 survey, we asked teachers to rate the extent to which the primary mathematics curriculum used at their schools was consistent with their "personal beliefs of effective teaching methods," "ways of teaching mathematics promoted in professional development sessions," and "the mission of the school," using a four point scale: 0= "not at all," 1= "to a small extent," 2= "to a moderate extent," and 3= "to a great extent." These three items loaded on one factor (eigenvalue=2.37, factor loadings= 0.87~0.91), so we aggregated to a composite variable by averaging the ratings over these three items (α=0.86).

Principals' expectations on high-quality mathematics instruction in 2009-10: On the 2009–2010 survey, we asked teachers to rate the extent to which their principal expected them to do the following things: "Have whole classroom discussion in which students explain how they solved tasks," "Have small-group discussion in which students explain how they solved tasks," "Use challenging, problem-solving tasks with my students," and "Use students' current mathematical thinking to inform my instruction." Teachers could rate their principals' expectations on a four-item scale: 0= "not at all," 1= "to a small extent," 2= "to a moderate extent," and 3= "to a great extent." Factor analysis showed these items described the same latent construct (eigenvalue=3.88, factor loadings=0.65~0.88), and thus we created a composite measure by averaging teachers' responses on these four items to indicate the extent to which teachers perceived principals' expectations for providing high-quality instruction (α=0.79).

Years taught mathematics in 2009-10: The relationship between experience and expertise is mixed. On the one hand, teachers with more experience accumulate more subject and pedagogical knowledge and skills from trial and error while teaching mathematics. Teachers can also gain knowledge of their students if they have taught the same grades. On the other hand, if experiences make teachers less flexible for different groups of students, or hesitant to respond to new instructional expectations, these experiences actually stymie gains in expertise (Borko & Livingston, 1989; Reynolds, 1992). To account for possible effects of teaching experience in our model, in the 2009–2010 survey, we asked teachers to fill in years in total, including 2010, that they taught mathematics.

Being certified to teach either middle or secondary grades in 2009-10: One indicator of whether the teacher has sufficient knowledge for teaching mathematics is whether this teacher is certified to teach this subject or grade (NCTAF, 1996). Thus, we denoted a teacher as "1" if he or she reported being certified to teacher either middle grades or secondary grades, and "0" otherwise.

Being a Caucasian: Prior studies showed that teachers' race affected their interactions with students, and assignment to an own-race teacher significantly increased mathematics and reading achievement of both Black students and White students (Dee, 2004). To control for teachers' racial effect on their content knowledge and instructional practices, we coded Caucasian as "1" and others as "0."

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